

Influence of Dust Charge Variations on the Dynamics of Lower Hybrid Waves in Streaming Dusty Plasma Systems

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ABSTRACT

The linear theory of excitation of electrostatic lower hybrid waves by streaming electrons is presented. The dispersion relation for the lower hybrid mode is obtained for a warm plasma containing electrons, ions and negatively charged dust grains with dust charge fluctuations. The frequency of the lower hybrid mode increases with the number density of negatively charged dust grains. It is found that the instability has the largest growth rate when the velocity of electron beam in the direction of the magnetic field is comparable to the parallel phase velocity of the mode. An increase in the dust population enhances the growth rate of the lower hybrid wave instability through the effect of capturing electrons. The dust charge fluctuations give rise to an additional unstable damping mode in plasma. However, the maximum growth rate decreases with an increase in the velocity of the streaming electrons travelling parallel to the magnetic field.

KEYWORDS: Lower hybrid, frequency, dispersion, growth rate.

I. INTRODUCTION

Lower hybrid waves, unlike other wave modes, can interact resonantly with electrons as well as ions. This allows lower hybrid waves to mediate the transfer of energy between the two plasma species and therefore lead to ion heating or electron acceleration, making them of considerable interest in many different laboratory [1,2] and space plasma environments [3-10]. The lower hybrid waves are generated by different mechanisms such as by electron or ion beams propagating in the plasma. A perpendicular ion-beam-driven lower hybrid mode has been observed by Chang [1]. Seiler and Yamada [2] have studied the excitation of lower hybrid wave instability by a spiraling ion beam in the linear Princeton Q-1 device. Papadopoulos and Palmadesso [3] have demonstrated that the lower hybrid modes can be generated by an energetic electron beam streaming through plasma along the magnetic fields. There has been a great deal of interest in studying the collective properties of dusty plasma [11-18], particularly for low frequency wave propagation. The dust charge may fluctuate due to a variety of reasons. Several authors have tried to incorporate the effect of these charge fluctuations on wave propagation and found interesting results [19-24]. Tsytovich [19] has considered a novel kinetic effect arising from inhomogeneities in dust charges and demonstrated collisionless drift wave instability. Mahanta *et al.* [20] have showed that dust charge fluctuation lead to damping of lower-hybrid like waves in cold dusty plasma.

In this paper, we study the excitation of lower hybrid waves by streaming electrons in magnetized dusty plasma. The streaming electrons propagating through the dusty plasma drive electrostatic lower hybrid waves to instability. In Sec. II, the instability analysis for lower hybrid wave excitation by streaming electrons is carried out. Results and discussions are given in Sec. III. Finally, the conclusion part is given in Sec. IV.

II. INSTABILITY ANALYSIS

We consider a collisionless and uniformly magnetized dusty plasma whose constituents are electrons, ions and negatively charged dust grains, with respective number densities n_{e0} , n_{i0} and n_{d0} , immersed in a static magnetic field B_s in the z -direction. The electrons are streaming along the magnetic field direction with a constant electron drift velocity $v_{e0} \hat{z}$. The quasineutrality condition at equilibrium is given by

$$en_{i0} = en_{e0} + Q_{d0}n_{d0}.$$

where $-e$ is the electronic charge, $-Q_{d0}$ is the dust grain charge and e is the positive ionic charge. Consider an electrostatic lower hybrid mode, propagating nearly perpendicular to the external magnetic field with propagation wave vector \mathbf{k} lying in the x - z plane. The equilibrium is perturbed by an electrostatic perturbation to the potential

$$\phi = \phi_0 e^{-i(\omega t - k_x x - k_z z)}. \quad (1)$$

The equation of motion, governing the perturbed velocity of plasma electrons is

$$m_e \left[\frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \mathbf{v}_e \right] = -e \mathbf{E} - \frac{e}{c} \mathbf{v}_e \times (\mathbf{B}_s + \mathbf{B}). \quad (2)$$

On linearizing Eq. (1), we obtain the perturbed electron velocities as

$$v_{1x} = -\frac{ek_x(\omega - k_z v_{eo})\phi}{m_e[(\omega - k_z v_{eo})^2 - \omega_{ce}^2]}, \quad (3)$$

$$v_{1y} = -\frac{iek_x \omega_{ce} \phi}{m_e[(\omega - k_z v_{eo})^2 - \omega_{ce}^2]}, \quad (4)$$

and

$$v_{1z} = -\frac{ek_z \phi}{m_e(\omega - k_z v_{eo})}, \quad (5)$$

where $\omega_{ce} = \frac{eB_s}{m_e c}$ is electron cyclotron frequency and subscript 1 refers to perturbed quantities. Substituting

the perturbed velocities given by Eqs. (3), (4) and (5) in the equation of continuity

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \quad (6)$$

we obtain the perturbed electron number density as

$$n_{e1} = -\frac{n_{eo} e \phi}{m_e} \left[\frac{k_x^2}{[(\omega - k_z v_{eo})^2 - \omega_{ce}^2]} + \frac{k_z^2}{(\omega - k_z v_{eo})^2} \right] \quad (7)$$

The perturbed ion density n_{i1} is obtained from Eq. (7) as,

$$n_{i1} = \frac{n_{io} e \phi}{m_i} \left[\frac{k_x^2}{\omega^2 - \omega_{ci}^2} + \frac{k_z^2}{\omega^2} \right]. \quad (8)$$

The dust response can be taken to be unmagnetized and is given as

$$n_{d1} = \frac{n_{do} Q_{do} \phi}{m_d} \left[\frac{k_x^2}{\omega^2} + \frac{k_z^2}{\omega^2} \right] \quad (9)$$

Now applying probe theory to a dust grain, the charge on a dust grain is known to be balanced with the plasma currents on the given surface. Following Prakash *et al.* [25], we obtain the dust charge fluctuation

$$Q_{d1} = \frac{i|I_{eo}|}{(\omega + i\eta)} \left(\frac{n_{i1}}{n_{io}} - \frac{n_{e1}}{n_{eo}} \right). \quad (10)$$

Substituting the value of n_{e1} and n_{i1} from Eqs. (7) and (8) in Eq. (10), we obtain

$$Q_{d1} = \frac{i|I_{eo}|}{(\omega + i\eta)} \left[\frac{e \phi}{m_i} \left(\frac{k_x^2}{\omega^2 - \omega_{ci}^2} + \frac{k_z^2}{\omega^2} \right) + \frac{e \phi}{m_e} \left\{ \frac{k_x^2}{[(\omega - k_z v_{eo})^2 - \omega_{ce}^2]} + \frac{k_z^2}{(\omega - k_z v_{eo})^2} \right\} \right] \quad (11)$$

Using Eqs. (7) – (9) and Eq. (11) in the Poisson's equation

$$\nabla^2 \phi = 4\pi n_{e1} - 4\pi n_{i1} - 4\pi n_{d0} Q_{d1} - 4\pi n_{d1} Q_{d0},$$

we obtain for $\omega_{ci} \ll \omega \ll \omega_{ce}$,

$$1 + \left(\frac{k_x^2}{k^2} \frac{\omega_{pe}^2}{(\omega - k_z v_{eo})^2 - \omega_{ce}^2} + \frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{(\omega - k_z v_{eo})^2} \right) (1 + \beta') + \left(-\frac{k_x^2}{k^2} \frac{\omega_{pi}^2}{\omega^2} - \frac{k_z^2}{k^2} \frac{\omega_{pi}^2}{\omega^2} \right) \left(1 + \beta' \frac{n_{eo}}{n_{io}} \right) - \frac{\omega_{pd}^2}{\omega^2} = 0 \quad (12)$$

where $\omega_{pe} = \left(\frac{4\pi n_{eo} e^2}{m_e} \right)^{1/2}$, $\omega_{pi} = \left(\frac{4\pi n_{io} e^2}{m_i} \right)^{1/2}$ and $\omega_{pd} = \left(\frac{4\pi n_{d0} Q_{d0}^2}{m_d} \right)^{1/2}$ are the electron, ion and

dust plasma frequencies, respectively, $\beta = \frac{|I_{eo}| n_{d0}}{e n_{eo}}$ is the coupling parameter given as $\beta = 0.1\pi a^2 n_{d0} v_{te}$,

$\eta = 0.01 \omega_{pe} \frac{n_{eo}}{n_{io}} \frac{a}{\lambda_D}$ is the time scale of delay, $C_s = \sqrt{\frac{T_e}{m_i}}$ is the ion acoustic speed, $\beta' = \frac{i\beta}{(\omega + i\eta)}$ and

$v_{te} = \left(\frac{2T_e}{m_e} \right)^{1/2}$ is the electron thermal velocity.

In deriving Eq. (12), we considered electrons as streaming and magnetized, ions as unmagnetized and nonstreaming, and dust grains as unmagnetized and nonstreaming. From Eq. (12), we obtain

$$(\omega^2 - \omega_1^2)(\omega + i\eta + i\beta_1 + i\beta_2) = -i\omega_1^2(\beta_1 + \beta_2), \quad (13)$$

where

$$\omega_1 = \omega_2 + \frac{1}{2K} \frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{(\omega - k_z v_{eo})^2} \omega \quad (14)$$

where $\omega_2^2 = \frac{1}{K} (\omega_{pi}^2 - \omega_{pd}^2)$

$$\beta_1 = \frac{1}{K} \left(\beta \left(-\frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{(\omega - k_z v_{eo})^2} \left(1 + \frac{\eta}{\beta} \right) + \frac{k_x^2}{k^2} \frac{\omega_{pe}^2}{\omega_{ce}^2} \right) + \frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{(\omega - k_z v_{eo})^2} i\omega \right), \quad (15)$$

$$\beta_2 = -\frac{\beta}{K} \frac{\omega_{pi}^2}{\omega^2} \frac{n_{eo}}{n_{io}}. \quad (16)$$

The dispersion relation given by Eq. (13) is a cubic equation in ω . The first bracket on the left hand side describes the lower-hybrid modes with dust particles and the second bracket indicates the pure unstable mode

due to the dust charge fluctuations. The term on the right hand side can be defined as a coupling term which is proportional to the sum of dust charge fluctuation and kinetic effect terms.

From Eq. (14), neglecting dust charge fluctuations and streaming of electrons, the dispersion relation of the lower hybrid mode with dust grains turns out to be

$$\omega_{lh}^2 = \left[\omega_{pi}^2 \left(1 + \frac{n_{eo}}{n_{io}} \right) - \frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{\omega^2} \right] / K. \quad (17)$$

A perturbative solution of Eq. (13) gives

$$\omega = \omega_1 - i(\beta_1 + \beta_2)/2. \quad (18)$$

The growth rate of the lower hybrid mode from Eq. (13) is given as

$$\gamma = \frac{\beta}{2k} \left(\frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pi}^2}{\omega^2} \frac{n_{eo}}{n_{io}} - \frac{k_x^2}{k^2} \frac{\omega_{pe}^2}{\omega_{ce}^2} \right) + \frac{\eta}{2k} \frac{k_z^2}{k^2} \frac{\omega_{pe}^2}{\omega^2}. \quad (19)$$

III. RESULTS AND DISCUSSION

We have used the following dusty plasma parameters in the present calculations: number density of plasma ions $n_{io}=5 \times 10^8 \text{ cm}^{-3}$, number density of dust grains $n_{do}=0.5 \times 10^4 \text{ cm}^{-3}$, guide magnetic field $B_s=0.1 \text{ G}$, 30 G and 320 G , mass of ion $m_i=39 \times 1836 m_e$ (Potassium-plasma), temperature of electron $T_e=5 \text{ eV}$, temperature of ion $T_i=0.2 \text{ eV}$, mass of dust grain $m_d=10^{12} m_i$ and dust grain radius $a=10^{-4} \text{ cm}$. The number of electrons which attach themselves with the dust grains in plasma is assumed to be 10^4 . We have discussed the application of the orbit motion limited (OML) theory to magnetized plasma in which lower hybrid waves propagate using standard model in which dust particles size and shape effects have been neglected. Such a model is reasonably valid in the limit $a \ll \lambda_{De} \ll \lambda$ (where a is the dust grain size, λ_{De} is the electron Debye length and λ is the wavelength of the fluctuation fields provided that the spread in Q_d/m_d (dust charge to mass ratio) for the dust particles in the equilibrium plasma may be neglected. Moreover, Jana *et al.* [21] have given the limits of the OML theory for magnetized dusty plasma, that the charging equation for dust grains can be valid if $a \ll \rho_L$ [where $\rho_L = \frac{v_{te}}{\omega_{ce}}$] is the electron Larmor radius, v_{te} is the electron thermal velocity and ω_{ce} is the electron cyclotron frequency. In the present calculations, we have chosen parameters in such a way that both the above mentioned conditions are satisfied.

The lower hybrid waves are said to be produced when $k_z/k \approx \sqrt{m_e/m_i}$ or $k_z/k \approx 0.00374$. Therefore, we concentrate in the region $k_z/k < 0.01$ for our numerical calculations and graph plotting. This region is important for lower hybrid waves with respect to ion heating^{18,19}.

Using Eq. (19), we have plotted in Fig. 1 the growth rate γ (rad/sec) of the unstable waves as a function of number density of dust grains $n_{do} \text{ (cm}^{-3}\text{)}$, for different values of electron streaming velocity, $k_z/k=0.004$ and external magnetic field of 0.1 G . From Fig. 1, it can be seen that the lower hybrid modes with electron streaming velocities $2 \times 10^4 \text{ cm/s}$, $4 \times 10^5 \text{ cm/s}$ and $1 \times 10^6 \text{ cm/s}$ show a growth. As the velocity is increased, the value of maximum growth rate decreases, as was observed by Prakash *et al.* [8]. A further increase in the beam velocity disables the streaming electrons to interact with the lower hybrid mode, and hence no growth is observed for velocities of $3 \times 10^7 \text{ cm/s}$ and $4 \times 10^7 \text{ cm/s}$.

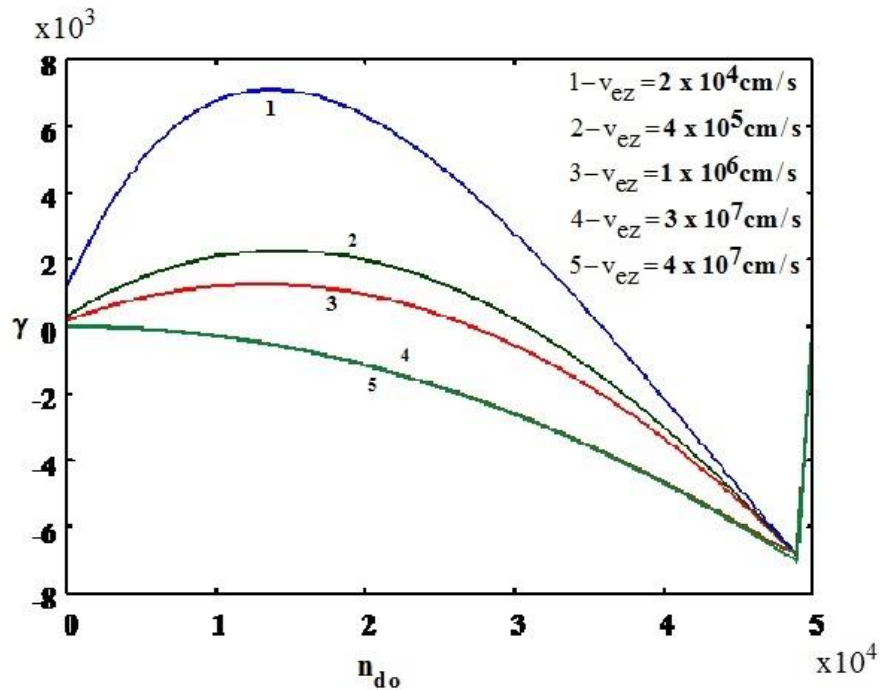


Fig. 1: Growth rate γ (rad/sec) of unstable mode as a function of dust grain number density n_{do} (cm^{-3}) for different electron streaming velocities and external static magnetic field $B = 0.1 \text{ G}$.

In Fig. 2 and Fig. 3, we have plotted the growth rate γ (rad/sec) of the unstable waves as a function of number density of dust grains n_{do} (cm^{-3}), for external magnetic field of 30 G and 320 G, respectively and electron streaming velocities of $1 \times 10^6 \text{ cm/s}$, $2 \times 10^7 \text{ cm/s}$, $4 \times 10^7 \text{ cm/s}$, $8 \times 10^7 \text{ cm/s}$ and $10 \times 10^7 \text{ cm/s}$. Rest of the plasma parameters remain same, as used for plotting Fig. 1. From Eq. (19), we find that the maximum growth rate occurs when ω_1/k_z is comparable to the electron streaming velocity. It may be noted from Fig. 2, that the maximum growth rate decreases with an increase in electron streaming velocity, for $B=30 \text{ G}$ as well as for $B=320 \text{ G}$. The growth rate has a maximum value at dust grain number density $n_{do} = 1.4 \times 10^4 \text{ cm}^{-3}$ for $B=0.1 \text{ G}$, at $n_{do} = 1.8 \times 10^4 \text{ cm}^{-3}$ for $B=30 \text{ G}$ and at $n_{do} = 2.5 \times 10^4 \text{ cm}^{-3}$ for $B=320 \text{ G}$. As the value of external static magnetic field increases, the frequency of lower hybrid wave increases (cf. Eq. (14)), and more number of dust grains are required for maximum growth.

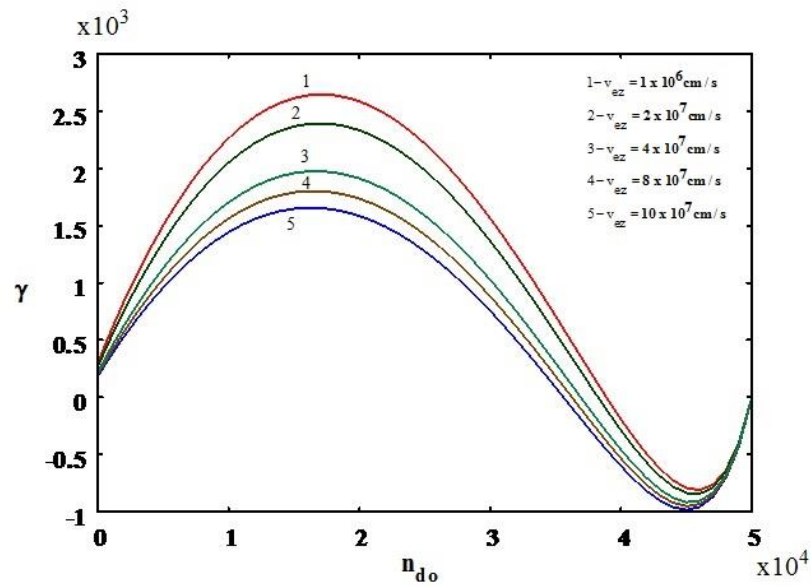


Fig. 2: Growth rate γ (rad/sec) of unstable mode as a function of dust grain number density n_{do} (cm^{-3}) for different electron streaming velocities and external static magnetic field $B = 30$ G.

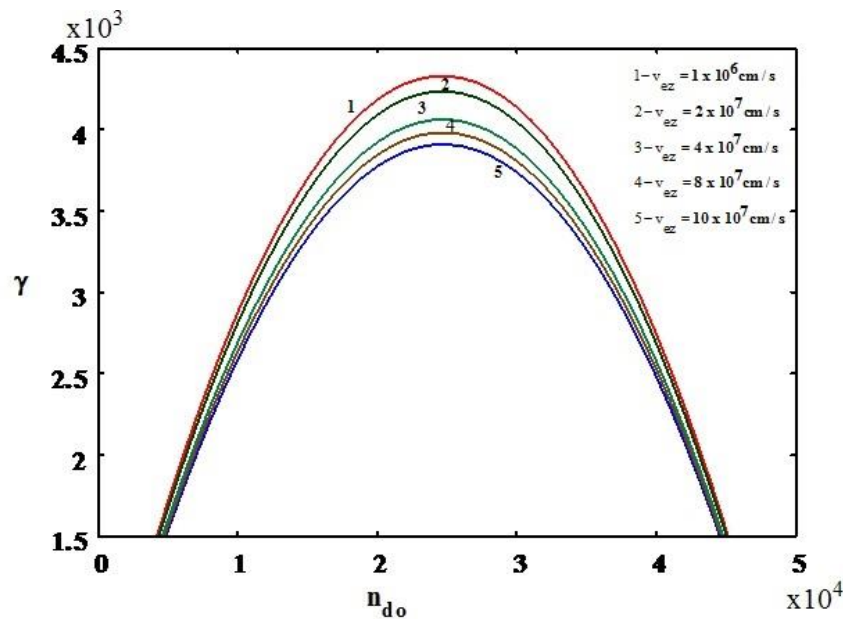


Fig. 3: Growth rate γ (rad/sec) of unstable mode as a function of dust grain number density n_{do} (cm^{-3}) for different electron streaming velocities and external static magnetic field $B = 320$ G.

Since the growth rate shows a maxima, when the parallel-wave phase velocity of the lower hybrid instability is approximately of the order of the electron streaming velocity, we expect efficient energy transport from the streaming electrons to the bulk of the ions and electrons via Landau damping, accelerating the electrons or heating the ions, necessary for thermonuclear plasma and fusion plasma.

The dust charging induces the existence of an additional short wavelength highly damped mode. In Fig. 2, the damping lower hybrid mode starts growing after $n_{do} = 4.5 \times 10^4 \text{ cm}^{-3}$. This indicates wave coupling of lower hybrid mode with the damping dust charge fluctuation mode. For large value of magnetic field, this dustcharge fluctuation mode is highly damped.

IV. CONCLUSION

The excitation of lower hybrid mode instability by streaming electrons in collisionless dusty plasma in the presence of dust charge fluctuations and external magnetic field has been studied. The frequency of the lower hybrid mode increases with an increase in the number density of negatively charged dust grains. It is found that the instability has the largest growth rate when the velocity of streaming electrons in the direction of the magnetic field is comparable to the parallel phase velocity of the mode. An increase in the dust population enhances the growth rate of the lower hybrid waves through the effect of capturing electrons. As the number density of negatively charged dust grains increases, the electron plasma density n_{e0} decreases with respect to ion plasma density n_{i0} . Thus, the ions have an effective mass that is less than m_i , and their greater mobility leads to increased wave generation.

However, the maximum growth rate decreases with an increase in the velocity of the electrons travelling parallel to the magnetic field. The presence of dust particles can modify the instability, as in the case of cometary emission [26]. Torney *et al.* [6] have studied the effect of dust grain population on the growth of the electrostatic waves and explained why no X-ray emission was detected from the optically bright, dusty comet Halebopp. The dust charging induces the existence of an additional short wavelength highly damped mode, which in the present

case corresponds to the mode with frequency $\omega = -i\eta \left(1 + \frac{\eta(\beta_1 + \beta_2)}{\omega_1^2} \right)$. The dust charging process introduces

new physics by modifying plasma dielectric properties.

V. REFERENCES

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