Analysis of Peristaltic Pumping of Non-Newtonian Hyperbolic Tangent Fluid Through a Porous Channel

Dr. Meenakshi Sharma
Assistant Professor, Department of Computer Science
University of Delhi, Delhi, India

ABSTRACT

The flow of peristaltic hyperbolic tangential fluid past porous walls with suction/injection is investigated. The regular perturbation method has been implemented to establish the appropriate expressions for velocity and pressure rise per wavelength by means of the assumptions of long wavelength and low valued Renolds number. The impact of numerous parameters on the pumping physical characteristics per wavelength is discussed graphically. It is recognized that the rising of the power law index of hyperbolic tangent fluid, rises the pressure in pumping region.

Keywords: *peristalsis, permeable walls, suction/injection.*

I. INTRODUCTION

Peristalsis occurs as in terms of progressive wave in which area contraction or expansion propagates along the tube. The natural property occurs in oesophagus, gastrointestinal tract, ductefferentus of the male reproductive tract, the bile duct, the fallopian tube, and the ureter. A few research works concerned with the peristaltic pumping can be seen in [1-7].

Among the non-Newtonian fluid models, the hyperbolic tangent fluid model [8] has the rheological behaviour of shear thinning. Nadeem and Akram [9] studied the peristaltic flow of such fluid through an asymmetric channel. Nadeem and Maraj [10] investigated the peristaltic pumping of a non-Newtonian hyperbolic tangent fluid in a curved channel with the application of Homotopy perturbation method. Saravana et al. [11] addressed the effects of peristalsis and elastic nature wall properties on the flow of conducting non-Newtonian hyperbolic tangent fluid with tapered channel with a standardized perturbation technique.

Motivated by the studies, the peristaltic transmission of a hyperbolic non-Newtonian tangent fluid with permeable wall is studied under long wavelength and small Reynolds number assumptions. The pressure rise/drop over one cycle of wave length are attained and the results are depicted graphically.

II. MATHEMATICAL MODEL

We consider the peristaltic wave propagation of a hyperbolic tangent fluid in a 2-dimensional symmetric uniform channel between two porous walls of constant speed 'c'. The fluid is blowing perpendicularly into the channel at the lower porous wall with fixed velocity $\bar{V}_0$ and is sucked out at the upper porous wall with same $\bar{V}_0$ as revealed in Fig.1 with the effect and impact of symmetric waves, it is adequate to study for mean width of the channel a.

$$H(X,t) = a + b\sin\frac{2\pi}{\lambda}(X - ct) \qquad \text{..............................(1)}$$

where b and λ denotes the amplitude and wave length respectively.

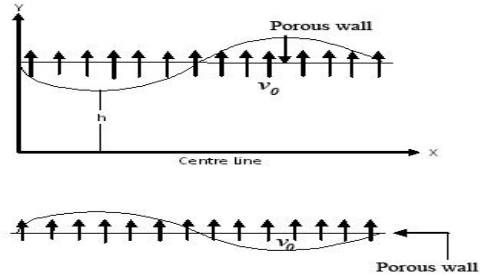

Fig. 1. Physical model

The respective conversion from the laboratory frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$x = X - ct,\ y = Y,\ u = U - c,\ v = V,\ p(x) = P(X,t)$ -----------------------(2)

where(u, v) and (U, V) are the velocity components, $p$andPare the corresponding pressures in the wave and fixed frames of reference respectively.

The non-dimensional quantities as given below:

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{a},\ \bar{u} = \frac{u}{c},\ \bar{v}_0 = \frac{v_0}{c},\ \phi = \frac{b}{a},\ \delta = \frac{2\pi a}{\lambda},\ \bar{p} = \frac{pa^2}{\eta_0 c\lambda},$$

$$h = \frac{H}{a},\ \bar{t} = \frac{ct}{\lambda}, \dot{\gamma}' = \frac{\dot{\gamma} a}{c},\quad \bar{\tau}_{xx} = \frac{a}{\eta_0 c}\tau_{xx}, \bar{\tau}_{xy} = \frac{a}{\eta_0 c}\tau_{xy},$$

$$\bar{\tau}_{yy} = \frac{a}{\eta_0 c}\tau_{yy},\ \mathrm{Re} = \frac{\rho ac}{\eta_0},\quad \bar{\gamma} = \frac{\gamma}{a},\ \bar{q} = \frac{q}{ac}, We = \frac{\Gamma c}{a}$$

The governing equations of flow field in wave frame analysis and with the assumptions of long wavelength and small Reynolds number is as follows

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} - k\frac{\partial u}{\partial y} \qquad \text{----------------------(3)}$$

where $\tau_{xy} = (1-n)\frac{\partial u}{\partial y} + nWe\left(\frac{\partial u}{\partial y}\right)^2$; $k = \mathrm{Re}.v_0$ in which v_0 is suction/ injection velocity

The volume flow rate $q$in an existing wave frame as in case of reference is given by

$$q = \int_0^{h(x)} u\,dy. \qquad \text{--------------------------(4)}$$

The instantaneous flux $Q(X, t)$ is

$$Q(X,t) = \int_0^h U\,dY = \int_0^h (u+1)\,dy = q + h.$$

The actual time average flux $\bar{Q}$ over a period $T(= \lambda/c)$ of the peristaltic wave is

$$\bar{Q} = \frac{1}{T}\int_0^T Q\,dt = \int_0^1 (q+h)\,dx = q+1. \qquad \text{-------------------------(5)}$$

III. PERTURBATION SOLUTION

The equation (2) is not a linear and the closed form solution may not be possible. Hence we apply the perturbation technique in terms of a small Weissenberg number. We expand u, p and q in terms of We as

$$u = u_0 + We\,u_1 + O\left(We^2\right), \qquad \frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + We\frac{\partial p_1}{\partial x} + O\left(We^2\right) \qquad \text{---------------------(6)}$$

$$q = q_0 + We\,q_1 + O\left(We^2\right)$$

The Zeroth and First order equations are

Equation of order We^0

$$\frac{dp_0}{dx} = (1-n)\frac{\partial^2 u_0}{\partial y^2} - k\frac{\partial u_0}{\partial y} \qquad \text{----------------------------(7)}$$

and the boundary conditions are

$$\frac{\partial u_0}{\partial y} = 0 \text{ at } \quad y = 0 \qquad \text{---------------------------(8)}$$

$$u_0 = -1 - \beta(1-n)\frac{\partial u_0}{\partial y} \text{ at } y = h \qquad \text{--------------------------(9)}$$

Equation of order We

$$\frac{dp_1}{dx} = (1-n)\frac{\partial^2 u_1}{\partial y^2} - n\frac{\partial}{\partial y}\left[\left(\frac{\partial u_0}{\partial y}\right)^2\right] - k\frac{\partial u_1}{\partial y} \qquad \text{----------------------------(10)}$$

and the boundary conditions are

$$\frac{\partial u_1}{\partial y} = 0 \text{ at } \quad y = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(11)}$$

$$u_1 = -\beta(1-n)\frac{\partial u_1}{\partial y} \text{ at } \quad y = h \qquad \text{-----------------------------(12)}$$

The Zeroth and First order Solutions are obtained by solving the resulting systems (7) and (10) with appropriate conditions (8-9) and (11-12)

$$u_0 = -1 - (1-n)p_0 e^{\frac{kh}{(1-n)}}\left(\beta + \frac{1}{k}\right) + \frac{p_0}{k}\left(\beta(1-n) + h\right) + \frac{(1-n)p_0}{k^2}e^{\frac{ky}{(1-n)}} - \frac{p_0}{k}y \qquad \text{---------------------(13)}$$

$$u_1 = p_1\left(\frac{\beta(1-n)}{k}\left(1 - e^{\frac{kh}{(1-n)}}\right) - \frac{(1-n)}{k^2}e^{\frac{kh}{(1-n)}} + \frac{h}{k}\right) + p_0^2\left(\begin{array}{l}\frac{4\beta n}{k^2}e^{\frac{kh}{(1-n)}} - \frac{3\beta n}{k^2}e^{\frac{2kh}{(1-n)}} - \frac{2\beta nhe^{\frac{kh}{(1-n)}}}{k(1-n)} - \\ \frac{\beta n}{k^2} + \frac{4n}{k^3}e^{\frac{kh}{(1-n)}} - \frac{n}{k^3}e^{\frac{2kh}{(1-n)}} - \frac{2nhe^{\frac{kh}{(1-n)}}}{k^2(1-n)}\end{array}\right) \qquad \text{-------(14)}$$

$$+\left(\frac{p_1(1-n)}{k^2} - \frac{4np_0^2}{k^3}\right)e^{\frac{ky}{(1-n)}} - \frac{yp_1}{k} + \frac{np_0^2}{k^3}e^{\frac{2ky}{(1-n)}} + \frac{2np_0^2 y}{k^2(1-n)}e^{\frac{ky}{(1-n)}}$$

$$\frac{\partial p_0}{\partial x} = \frac{q_0 + h}{c_1} \qquad \text{--------------------------(15)}$$

$$\frac{\partial p_1}{\partial x} = \frac{q_1}{c_1} - \left(\frac{\partial p_0}{\partial x}\right)^2\frac{c_2}{c_1} \text{ ----------------- (16)}, \quad q_0 = \int_0^h u_0\,dy \qquad \text{-------------------------(17)}$$

$$q_1 = \int_0^h u_1 dy \quad \text{----------------------(18)}$$

Where $c_1 = -\frac{\beta h(1-n)}{k} e^{\frac{kh}{(1-n)}} + \frac{\beta h(1-n)}{k} - \frac{(1-n)}{k^2} e^{kh} + \frac{(1-n)^2}{k^3} e^{\frac{kh}{(1-n)}} - \frac{(1-n)^2}{k^3} + \frac{h^2}{2k}$

$$c_2 = \frac{4\beta hn}{k^2} e^{\frac{kh}{(1-n)}} - \frac{3\beta hn}{k^2} e^{\frac{2kh}{(1-n)}} - \frac{2\beta h^2 n}{k(1-n)} e^{\frac{kh}{(1-n)}} - \frac{\beta hn}{k^2} + \frac{4hn}{k^3} e^{\frac{kh}{(1-n)}} - \frac{hn}{k^3} e^{\frac{2kh}{(1-n)}} - \frac{2nh^2}{k^2(1-n)} e^{\frac{kh}{(1-n)}}$$
$$- \frac{6n(1-n)}{k^4} e^{\frac{kh}{(1-n)}} + \frac{n(1-n)}{2k^4} e^{\frac{2kh}{(1-n)}} + \frac{2hn}{k^3} e^{\frac{kh}{(1-n)}} + \frac{11n(1-n)}{k^4}$$

Substituting (15) and (16) into the equation (6), using $\frac{\partial p_0}{\partial x} = \frac{\partial p}{\partial x} - We \frac{\partial p_1}{\partial x}$ and neglecting $O(We^2)$, we get

$$\frac{dp}{dx} = \frac{q+h}{c_1} - We(q+h)^2 \frac{c_2}{c_1^3} \quad \text{---------------------- (19)}$$

The dimensionless pressure rise per wavelength in the wave frame of reference is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad \text{------------------------(20)}$$

IV. RESULTS AND DISCUSSION:

The variation of pressure rise with $\bar{Q}$ for distinct values of Weissenberg number We is depicted in Fig. 2. From the figure, we noticed that the larger the We, the greater the pressure growth in the pumping region and in the co-pumping region. The difference of Δp with $\bar{Q}$ for power-law index number is depicted in Fig.3. In the figure, we observe that larger the power-law index number n the greater the pressure rise for the pumping region and opposite trend is found in the co-pumping region.

From Fig. 4, we find that the rising of the velocity slip parameter β, dropping the pressure growth against which the pump works when $\bar{Q} < 0.5$ and the reverse trend is observed from that point $\bar{Q} = 0.5$ onwards. In Fig.5, we notice that the pumping curves intersects at $\bar{Q} = 0.45$. For $\bar{Q} < 0.45$, the greater the suction/ injection parameter k, pressure rise decreases against which the pump works but the trend is reversed for $\bar{Q} > 0.45$.

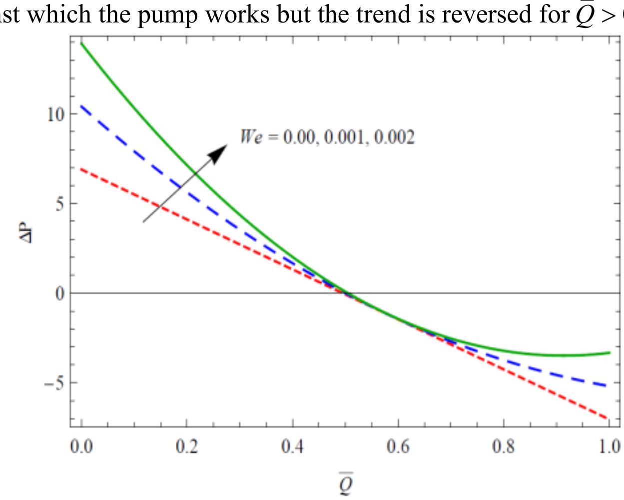

Fig. 2. ΔP *vs* $\bar{Q}$ ***for*** We ***with*** $\phi = 0.6$, $n = 0.06$, $\beta = 0.01$ ***and*** $k = 1$

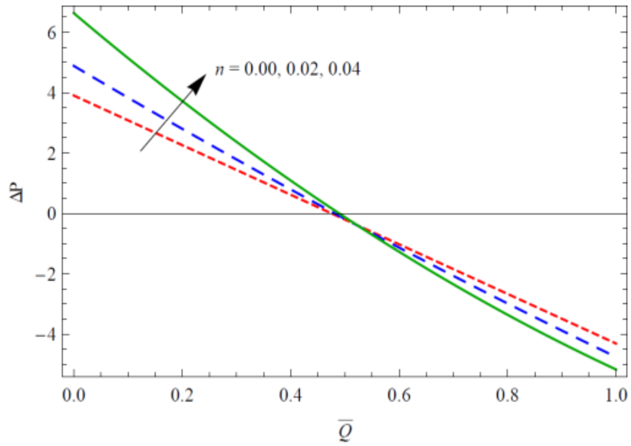

Fig. 3. ΔP vs $\bar{Q}$ ***for*** n ***with*** $\phi = 0.6$, $We = 0.001$, $\beta = 0.01$ ***and*** $k = 1$

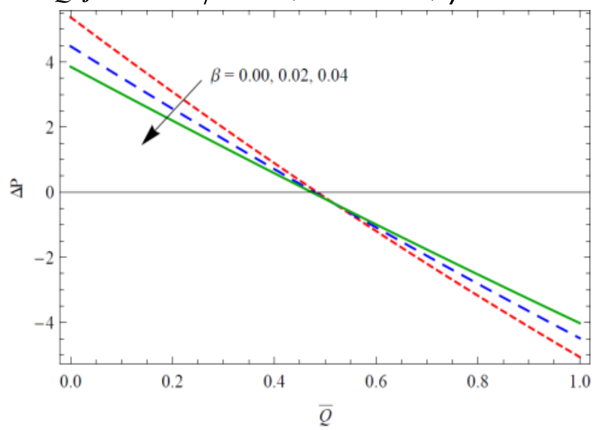

Fig. 4. ΔP vs $\bar{Q}$ ***for*** β ***with*** $\phi = 0.6$, $We = 0.001$, $n = 0.02$ ***and*** $k = 1$

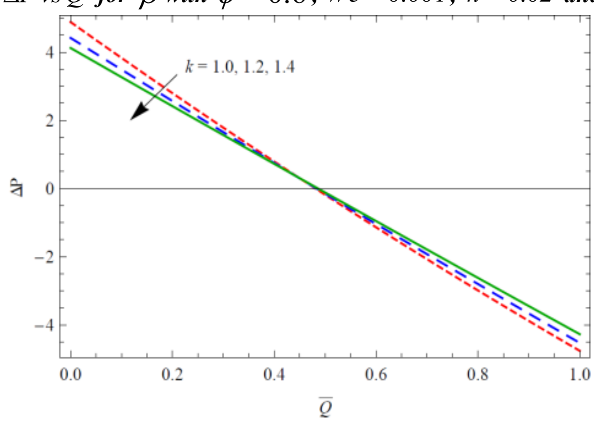

Fig. 5. ΔP vs $\bar{Q}$ ***for*** k ***with*** $\phi = 0.6$, $We = 0.001$, $n = 0.02$ ***and*** $\beta = 0.01$

REFERENCE

1. *Shapiro, A.H., Jaffrin, M.Y., Weinberg, S.L. "Peristaltic pumping with long wavelengths at low Reynolds number". J. Fluid Mech.**37** (1969) 799-825.*
2. *Saravana, R., Sreenadh, S., Venkataramana, S., Hemadri Reddy, R., Kavitha, A. Influence of slip conditions, wall properties and heat transfer on MHD peristaltic transport of a Jeffrey fluid in anon-uniform porous channel. International Journal of Innovative Technology and Creative Engineering, **1(11)** (2011) 10-24.*
3. *Vajravelu, K.,Sreenadh, S.,Saravana, R. Combined influence of velocity slip, temperature and concentration jump conditions on MHD peristaltic transport of a Carreau fluid in a non-uniform channel. Applied Mathematics and Computation, **225**(2013) 656-676.*

4. *Kumar, M.A., Sreenadh, S., Saravana, R, Reddy, R.H., Kavitha, A. Influence of velocity slip conditions on MHD peristaltic flow of a Prandtl fluid in a non-uniform channel, Malaysian Journal of Mathematical Sciences,* ***10(1)****, (2016) 35–47.*
5. *HariPrabakaran, P., Hemadri Reddy, R.,Saravana, R Kavitha, A and Sreenadh, S. Influence of elasticity on MHD peristaltic transport of a Jeffrey fluid through porous medium channel with heat and mass transfer, Advances and Applications in Fluid Mechanics, 17(1), 2015, 1-16.*
6. *HariPrabakaran, P., Kavitha, A., Saravana, R., Hemadri Reddy, R., Sreenadh, S. Peristaltic transport of a Fourth grade fluid between porous walls with suction and injection", International Journal of Pure and Applied Mathematics, 86(2), 2013, 293-300.*
7. *HariPrabakaran, P., Hemadri Reddy, R., Sreenadh, S., Saravana, R. Kavitha, A. "Peristaltic pumping of a Bingham fluid in contact with a Newtonian fluid in an inclined channel under long wave length approximation", Advances and Applications in Fluid Mechanics, 13(2), 2013, 127-139..*
8. *Pop, I., Ingham, D.B. "Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media". Pergamon, Amsterdam, NewYork (2001).*
9. *Nadeem, S., Akram, S. "Peristaltic transport of a hyperbolic tangent fluid model in an asymmetric channel". Zeitschrift fur Naturforschung, 64a (2009) 559–567.*
10. *Nadeem. S., Maraj. M.N. "The Mathematical Analysis for Peristaltic Flow of Hyperbolic Tangent Fluid in a Curved Channel. Communications in Theoretical Physics", 59(6) (2013) 729. doi:10.1088/0253-6102/59/6/14.*
11. *Saravana. R, Reddy. RH, Goud. JS, Sreenadh. S. "MHD peristaltic flow of a hyperbolic tangent fluid in a non-uniform channel with heat and mass transfer", IOP Conference Series: Materials Science and Engineering 263 (6), 062006. doi:10.1088/1757-899X/263/6/062006 pp. 850–861.*