Investigating the Role of Matrix Models in Describing Energy-Momentum and Their Statistical Mechanical Implications

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In a series of notes [2] - [6], it was shown how a 2x2 energy matrix eigenvalue problem is contained in Einstein´s 1905 energy momentum equation $E^2=p^2 + m^2$ (c=1). (1) This model used linear algebra and gave rise a velocity matrix V and acceleration matrix dV/dt as well as to an energy matrix which was able to provide time evolution through exp(-iHt). Thus, quantum mechanical formalism naturally appeared in this model, with no assumptions being made. From this formalism, it was possible to calculate zittergwegung of V(t), dV(t)/dt and X(t) and obtain results presented in the literature [1]. It was further argued in [6] that at the root of this model there seemed to be a ¨bouncing photon¨ i.e. a physical resonance even with p=0. The zitterbewegung then gave various types of periodic motion over a sphere and seemed to give physical justification for a wavelength.

The purpose of this note is to see whether the resonance picture (bouncing photon) of a particle can in any way be linked to statistical mechanics.

Knuth Albany Model

In 2014, K. Knuth [15] proposed a probabilistic model to describe a relativistic particle moving forward with speed v in one dimension as having probabilities for moving forward and backward. He gives these probabilities as .5(1+v) and .5(1-v). These are exactly the probabilities found in the 2x2 matrix model of [2]-[6]. The positive energy eigenvector is 1/sqrt(2) (sqrt(1+v) , sqrt(1-v)). Taking the norm one finds: .5(1+v) + .5(1-v) i.e. the two probabilities add to one.
The 2x2 matrix model, however, gives a second solution, name that for -E which also represents probabilities.

It is interesting to note, however, that when working with the energy eigenvector, one is dealing with square roots of probabilities. It is only when taking expectation values of norms that one sees the probabilities explicitly. This is reminiscent of the wavefunction in quantum mechanics. In the 2x2 model, it is suggested that there is an underlying physical reality to the eigenvector, namely that it is associated with a bouncing photon. Thus, the ¨square root of the probability¨ is not just a mathematical curiosity.

We further check if the probabilities maximize an entropy expression S=-P1 ln(P1) - P2 ln(P2) subject to the velocity matrix constraint $\langle +E| V |+E\rangle= v$ where $V=\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$ i.e. P1 - P2 = v.

If one maximizes S + w(P1 - P2), with w being a Lagrange multiplier, one finds that: ln{ (1-v)/(1+v)} = 2w and
P1= exp(-w)/ {exp(-w) + exp(w)} and P2= exp(w)/{exp(-w) + exp(w)} which does lead to P1=(1+v)/2 and P2=(1-v)/2 so this distribution does appear to maximize the entropy. This is interesting because not only is 1/sqrt(2) (sqrt(1+v), sqrt(1-v)) an energy eigenvalue of the 2x2 model, but it is also the square root of a probability distribution which has a maximum entropy for the constraint P1-P2=v. This constraint is built into the 2x2 model.

It also interesting to note that in the 2x2 model, the constraint < Vmatrix > =v suffices to yield solutions for P1 and P2 because the constraint is equivalent to P1-P2=v and P1+P2=1 by normalization. No maximization of any entropy statement is needed.

We argued in [2]-[6] that this 2x2 model which naturally appears in the energy momentum equation (1) is consistent with quantum mechanical formalism. It also appears to have a link to statistical mechanics. It appears that the eigenvector of the 2x2 matrix model which uses quantum mechanical formalism is equivalent to the maximizing of entropy. In other words, if the time evolution operator exp(-iHt) operating on a vector of square roots of the probabilities only changes it by a phase, then maximum entropy has been reached.

One may ask whether the entropy associated with the bouncing photon disrupts the Maxwell-Boltzmann distribution. One can try to solve for a distribution which maximizes entropy subject to constraints. Now, P(v) is replaced with P1(v) and P2(-v) such that P1+P2=f(v) and P1-P2=vf(v). Then the pieces related to P1 and P2 in the equation to be maximized (see [17] for the maximization procedure) are:

P1 ln(P1) + P2 ln(P2) + w(P1-P2) - y(P1+P2) e where w and y are Lagrange multipliers and e is the energy associated with v. Thus P1 is proportional to exp(-w-ye) and P2 to exp(w-ye).
Then, (P1-P2)/(P1+P2) = v = {exp(-w)-exp(w)}/{exp(-w)+exp(w)}.
This leads to the entropy of Knuth, so the overall entropy for the forward and backward motion is P1f(v) and P2f(v), where f(v) is the Maxwell-Boltzmann distribution again.

sqrt(1-v^2) Operator

In [2]-[6] a matrix |0 1| which represents sqrt(1-v^2) was largely ignored. We wish to examine it
|1 0|

here briefly, because of its connection to energy. We note that:

[H, sqrt(1-v^2)] = 2p |0 -1| where in [2]-[6] dV/dt had been associated with 2m|0 -1|
|1 0| |1 0|

Now d sqrt(1-v^2)/dt = -1/m p dv/dt, so this appears to be consistent with [H, O]= dO/dt.

Furthermore, sqrt(1-v^2) will have zitterbewegung which is interesting because it is the reciprocal of the energy term. This energy term was said to account for a physical frequency. Perhaps the zitterbewegung of sqrt(1-v^2) represents its physical aspects.

exp(iHt) |0 1| exp(iHt) = |0 1| + i cos(Et)sin(Et) 2p |0-1| + 2sin2(Et)2cos(a){sin(a)V - cos(a)|0 1|}
           |1 0| |1 0| |1 0| |1 0|

Here, V is the velocity matrix |1 0| and |0 -1| is related to dV/dt.
                                            |0 -1| |1   0|

2x2 Matrix Model and Counting states

In nonrelativistic statistical mechanics, one must count energy states. Phase space consisting of p and x is used to do this because in a one dimensional well it is p and not E that is equally spaced. Simply using dE as a measure would not count energy states, but dp does. In the nonrelativistic situation p=mv and so both p and v essentially count states. Such is not the case in the relativistic case. One may ask how the 2x2 model allows for the counting of states as one has the parameters E, p and v. Which should be used: dE, dp or dv?

First of all, the energy eigenvalue equation for the 2x2 model is:

|p   m| |+E>   = E |+E>
|m -p |

Here, one can notice two things. First of all, the energy matrix is linear in p and so suggests that moving from one energy matrix to another means changing p to p+dp. This suggests p is the measure. Secondly, p and E are both linear in this equation. This is important if one notes that zitterbewegung calculations suggest 2*3.14* E is a frequency, i.e. it is associated with 1/time. This immediately suggests a possible wave equation with:
Vphase = E lambda where lambda is the wavelength

If such a particle is restricted to a space of length L and one wants the time to be an integer n times the frequency then: L= vphase * n * 2 3.14/E. Making this restriction suggests that the frequency is physical as the zitterbewegung calculations of [2]-[6] suggest.
Then lambda = L/ (2*3.14 n). Now, p=Ev, so if E is like a frequency (i.e. 1/time), p is like this frequency dragged through space, so one may suggest p is proportional to 1/lambda. This is in keeping with p and E being linear in the eigenvalue equation. In such a case, one changes from one p to another in increments of 2*3.14/L, so dp counts states in one dimension.

Statistical Mechanics

The statistical mechanical probability weight, proportional to exp(-E/T), is important in calculations. In general, one has particles colliding either with themselves or the walls of box. The box is at a temperature T, related to the average kinetic energy, but T can also be thought of as related to a Lagrange multiplier which ensures there is a total amount of energy. At first glance, it might appear that random or disordered collisions are occurring, but it is argued that this is not the case. The system appears to be very ordered, with constant temperature, pressure and density throughout and with ordered sound waves stirred up by perturbations Microscopically, there is a very specific momentum distribution. (It should be noted that this distribution is often assigned to the Central Limit Theorem in the literature.) It almost appears as if the system is in a state of resonance both macroscopically and microscopically. If one were to introduce a particle into this equilibrium system, it would not undergo random collisions because the Maxwell-Boltzmann distribution dictates the probabilities of the momenta of the particles with which it will collide. If one examines the Maxwell-Boltzmann distribution f(p) in both relativistic or non-relativistic cases, one finds:

df(p)/dp = -dE/dp (1/T) f(p) (Note: dE/dp=v in both relativistic and nonrelativistic cases)

f(p) is the probability for a momentum p, so df/dp is the change in probability with a change in p.

Thus, it appears as if dE/T, which arises from a constraint on energy in maximization of entropy calculations, is the difficulty of a particle obtaining a dE amount of energy in a system of temperature T. The bigger the energy the more difficult, hence the decrease of f(p). This is interesting as statistical mechanics seems to driven by quantum mechanical momentum as a counter of the number of states in an energy interval. In quantum mechanics in one dimension, momentum p = hbar n 2*3.14/L. Thus, p is equally spaced although the energy between levels increases. For a large p, moving from one p level to another increases the momentum by a constant dp, but dE is much larger than dE for a small p. Thus, dE/T is larger and the transition is more difficult.

The question arises as to whether these ideas can be at linked at all to the 2x2 matrix model. A particle with p=0 (i.e. at rest) has an energy eigenvector of (1,1) which is equivalent to equal weights of the two velocity eigenvectors (1,0) and (0,1). A dV/dt operator is crucial in transitioning between the two eigenvectors.

For p>0, the +E energy eigenvector becomes (cos(a/2), sin(a/2)), but there are still components of the two velocity eigenvectors with weights cos(a/2) = sqrt(1+v) and sin(a/2)=sqrt(1-v). A dV/dt operator is still in the picture. It is this same dV/dt operator that appears in transitioning from one energy to another as can be seen from:

$$\begin{pmatrix}\cos(a/2 + d)\\ \sin(a/2+d)\end{pmatrix} = \begin{pmatrix}\cos(a/2)\\ \sin(a/2)\end{pmatrix} + d \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} \begin{pmatrix}\cos(a/2)\\ \sin(a/2)\end{pmatrix}$$

This might suggest that the resonance mechanism which seems to exist within the particle is the same mechanism used to accelerate from one energy or momentum to another.

The 2x2 model has a +E eigenvector of (cos(a/2), sin(a/2)). The angle a/2 moves from 45 degrees where the vector is 1/sqrt(2) (1,1) to a point where v/c=1. Let one assume that for every region da, there are an equal number of energy eigenvectors (of different energies). Then: p/E=cos(a) so:

dp= -E*E da.

Now dp=constant because momentum levels in a 1-D well are equally spaced. Thus, da decreases as E increases, so for high p, there are very few eigenvectors (cos(a/2), sin(a/2)) in the da region. It seems that this may already suggests a lower chance of populating higher p states, even without a temperature constraint being imposed. This suggestion only considers states available to a single particle. The usual argument used to obtain exp(-E/T) is that if a large amount of energy is assigned to one particle and E is fixed, there is less energy for the rest of the particles and so fewer state arrangements, hence a lower probability.

Zitterbewegung and Statistical Fluctuations

In the energy momentum equation (1), it is implied that there is a velocity parameter. The 2x2 matrix model which is contained in the equation, however, leads to velocities to the right and left with speed c, such that the expectation value is v. Thus, ¨v¨ is not the full velocity picture. In notes [2]-[6], it was shown how zitterbewegung results could be written for V(t), dV(t)/dt and X(t) showing various periodic or circular motions over a sphere. These were governed by terms such as cos(2Et). If an expectation value was taken with respect to the +E eigenvector, the zitterbewegung motion disappeared leaving only the parameter v for the case of V(t). Similar, results held for other quantities. The Maxwell-Boltzmann distribution, however, is based entirely on the parameter v. If zitterbewegung is physical, however, its motion, it seems, should give rise to fluctuations in the Maxwell-Boltzmann results.

In addition, in statistical mechanical calculations the deBroglie wavelength enters when one has interparticle distances which are very small. The zitterbewegung calculations may account for this as X(t) has a distance range in which there are various periodic motions over a sphere. Thus, within such a range it may not be accurate to consider the particle as moving with a velocity v.

Finally, it can be noted that Burra Gautam Sidharth, in his book The Thermodynamic Universe: Exploring the Limits of Physics [26] notes:

¨We have in effect equated the statistical fluctuations when there are N particles to the Quantum Mechanical fluctuations. The former fluctuations take place over over a scale R/sqrt(N) where R is the size of the system of particles and N the number of particles in the system. The quantum mechanical fluctuations take place at a scale of the order of the Compton wavelength.¨

The Compton wavelength appears in Zitterbewegung calculations.

Conclusion

In conclusion, the 2x2 model seems to suggest using dp (change in momentum) as a constant when counting energy levels in statistical mechanical calculations of the partition function. We have also examined further links of the model to the de Broglie wavelength which is used to justify the use of equally spaced dp for energy state counting. It also appears that a 2x2 matrix model is consistent with Knuth´s statistical model. The two component positive energy eigenvector represents square roots of the forward and backward motion probabilities. These probabilities maximize entropy. Thus, it appears that the 2x2 model, which uses quantum mechanical formalism, is also equivalent to a statistical mechanical model. In other words, at least in this model, there appears to be a link between statistical mechanics and quantum mechanics which is interesting. It should be noted, however, that in this note very little has been shown about this link other than the equivalency of the energy eigenvector with maximum entropy. It is also suggested that zitterbewegung could lead to statistical fluctuations and that they may account for issues in statistical mechanics when interparticle separations reach the size of the de Broglie wavelength.

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