Exploring Zitterbewegung and DeBroglie Wavelengths through a 2x2 Matrix Model of Relativistic Momentum and Energy

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In a series of notes [2]-[7], it was seen that a 2x2 matrix eigenvalue energy equation could be obtained from Einstein´s 1905 equation $E^2=p^2+m^2$ (1), together with an energy matrix H, a velocity matrix V and an acceleration matrix dV/dt. A quantum mechanical type formalism appeared without any assumptions of quantum mechanics. It was argued that this formalism could be applied to quantum mechanical as well as classical problems. It was seen that exp(-iHt) could serve as an evolution operator and that the regular equations of zitterbewegung [1] followed.
The zitterbewegung equations are important because they predict periodic motion for position, velocity and acceleration matrices. A first objective of this note is to try to examine the root of this periodic motion in the 2x2 model and a second objective is to see how the De Broglie wavelength is associated with the model.

De Broglie Wavelength

The deBroglie wavelength of a particle, which is proportional to 1/momentum, has very important physical consequences. It is not only part of quantization (for example, an integral number of half-wavelengths must fit inside a one dimensional well), essential for wave functions and scattering phenomena, but also has an important effect on statistical mechanics. For a one dimensional potential well of length L, the momentum is quantized and is proportional to n. Thus, there there is equal spacing of momentum levels, but not energy levels. Thus, statistical counting of states is based on momentum counting. In 3 dimensions, one uses dpx dpy dpz. If one wishes to use dE, an energy level density function is required.

Statistical Method of A. Niehaus

A. Niehaus [13] has recently developed a statistical model for the electron which does not use quantum mechanics, but rather depends on a probability distribution function. He uses zitterbewegung as well ¨instantaneous angular momentum changes¨ not predicted by zitterbewegung and develops graphs of probability distributions for the electron. He considers spin and the deBroglie wavelength as half the radius of an emerging toroidal distribution. He finds that the zitterbewegung motion is confined to a three dimensional sphere. This approach seems very interesting as 2x2 matrix approach of [2]-[7] also does not depend on quantum mechanics (although the same math appears). It too predicts zitterbewegung motion on a three dimensional sphere if one consider oscillations perpendicular to the one dimensional particle motion to be a shadow of circular motion.

Special Relativistic Consequences

An immediate question which arises is why a two dimensional matrix model is needed to describe the one dimensional motion of a particle. A second question is why there should be any ¨frequency¨ in this problem whatsoever. In special relativity, one has a two dimensional right angle triangle, with E as the hypotenuse, p as the adjacent (c=1 here) and m as the opposite. One can scale this triangle by dividing each side by E. Let us call the angle between the hypotenuse and the adjacent a1. If a particle accelerates, i.e. changes to a new velocity, it will have a new triangle with a new angle a2, but the hypotenuse will still be 1. Thus, the second state is a rotated form of the first. As a consequence, special relativity leads one to consider one dimensional motion and acceleration of a particle as a two dimensional rotation. By introducing a 2x2 matrix approach to the energy momentum equation (1), an extra idea enters which may provide a physical basis for the frequency of zitterbewegung.

Dynamics of the Energy Matrix H

A 2x2 energy matrix approach to the energy momentum equation (1) was developed by noting that p/E=cos(a) could be written as:

| cos(a/2), sin(a/2)| V |cos(a/2)| with V= |1   0|
|sin(a/2) | |0 -1|

It was also noted in [2] - [7] that [cos(a/2), sin(a/2)] is proportional to [sqrt(1+v), sqrt(1-v)] and that a physical picture of a bouncing signal photon could be applied to the 1+v and 1-v terms. The V matrix was seen to independent of v and to contain two eigenvalues +/-1 which were said to be associated with forward and backward motion. It is perhaps the case, that this backward and forward motion is specifically associated with a bouncing photon. In other words, the entire 2x2 matrix model is based on the idea of sqrt(1+v) and sqrt(1-v) as being the ¨drivers¨ of the model. This bouncing photon provides a physical basis for a ¨physical frequency¨ in the problem, it is no longer a ¨physical picture that may possibly be applied to the terms sqrt(1+v) and sqrt(1-v)¨. The sqrt(1+v) and sqrt(1-v) terms are proportional to cos(a/2) and sin(a/2), so the linear problem of a particle moving from one velocity to another is now a problem of a rotation through a half angle a/2. In fact, the energy momentum equation can be written as an energy eigenvalue problem:

In the 2x2 matrix method, an energy matrix H = |p m| is used. A velocity matrix V=|1 0|
|m-p| |0-1|
Is also employed. It was shown in [2]-[7] that |cos(a/2), sin(a/2) | was a positive energy eigenvector with cos(a)=v=p/E. This eigenvector is also proportional to (sqrt(c+v), sqrt(c-v)). We set c=1. The velocity matrix has a +1 eigenvector (1,0) and a -1 eigenvector (0,1). This was

interpreted as meaning that there existed forward and backward motion. Specifically, this seems to imply that there is a photon bouncing back and forth. The expectation value of the V operator is v for a moving particle, but there is no sign of ¨v¨ in the velocity matrix or its eigenvectors.

If one solves the eigenvalue problem for H with p=0 one finds eigenvalues of +/- m and a positive energy eigenvector of (1,1) which is comprised of the positive and negative velocity eigenvectors. For p>0, there is still an internal forward and backward motion (as well as motion all over a three dimensional sphere).

It was already seen in [2]-[7] that:

$$\left|\begin{matrix}\cos(a/2 + d)\\ \sin(a/2 + d)\end{matrix}\right| = \left|\begin{matrix}\cos(a/2)\\ \sin(a/2)\end{matrix}\right| + d\left|\begin{matrix}-\sin(a/2)\\ \cos(a/2)\end{matrix}\right|$$ where d is infinitesimal.

The first vector on the right is the +E eigenvector and the second the -E eigenvector. The two are perpendicular. An acceleration rotates one along a unit circle and so the tangent, is perpendicular to the radius. It was shown in [2]-[7] that the negative energy eigenvector can be written as (2)

$$\left|\begin{matrix}0 & -1\\ 1 & 0\end{matrix}\right| \left|\begin{matrix}\cos(a/2)\\ \sin(a/2)\end{matrix}\right|$$ which is a matrix that multiplies the +E eigenvector. This matrix should somehow be associated with an acceleration matrix as acceleration is

occurring as one changes from a/2 to a/2+d. In particular, consider the velocity matrix and its two eigenvectors (1,0) with a +1 eigenvalue and (0,1) with a -1 eigenvalue. If one asks for a matrix that will describe an ¨acceleration” from one eigenvector to the other, one might expect:

$$2\left|\begin{matrix}0 & -1\\ 1 & 0\end{matrix}\right|$$ proportional to dV/dt. Thus, the matrix in (2) does seem to be directly related to an acceleration matrix.

Moving from a/2 to a/2+d is, however, also a rotation which should be described by:

$$R=\left|\begin{matrix}\cos(d/2) & -\sin(d/2)\\ \sin(d/2) & \cos(d/2)\end{matrix}\right| = \left|\begin{matrix}1 & 0\\ 0 & 1\end{matrix}\right| + d/2 \left|\begin{matrix}0 & -1\\ 1 & 0\end{matrix}\right|$$ Again, the acceleration type matrix appears.

The energy matrix $H/E = \left|\begin{matrix}\cos(a) & \sin(a)\\ \sin(a) & -\cos(a)\end{matrix}\right|$ transforms as $R^{-1}HR$, where R is the rotation matrix,

but the rotation is through a half angle d/2.

$$H/E(a+d) = H/E(a) + d\left|\begin{matrix}-\sin(a) & \cos(a)\\ \cos(a) & \sin(a)\end{matrix}\right| = R^{-1}HR$$ and R^{-1} is the transpose of R.

It was also noted in [2]-[7], that (3) [H, V] is proportional to dV/dt. Thus, this acceleration matrix is linked to H, the energy matrix and V the velocity matrix. In fact, a solution to (3) is V(t)=exp(iHt) V exp(-iHt) which is identical to the quantum mechanical result [1]. Thus, the H/E operator evolves states through time. It was shown in [2]-[7] that exp(-iHt) could in fact rotate any unit vector (cos(na), sin(na)) into a vector proportional to (cos(a/2), sin(a/2)), the +1 eigenvector of H/E.

These relations are examined in detail because exp(-iHt) is the source of periodic motion in zitterbewegung calculations, but it seems to have its roots in the (sqrt(1+v), sqrt(1-v)) eigenvector of the 2x2 matrix model, with the root of the periodic motion being the bouncing photon associated with the two eigenvectors of the velocity matrix.

As a further example, let us consider in more detail the action of the H matrix on its eigenvector which we will call (a,b).

$\begin{vmatrix} p & m \\ m & -p \end{vmatrix} \begin{vmatrix} a \\ b \end{vmatrix} = H a \begin{vmatrix} 1 \\ 0 \end{vmatrix} + H b \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} pa \\ ma \end{vmatrix} + \begin{vmatrix} mb \\ -pb \end{vmatrix}$ a is proportional to sqrt(c+v) and b to sqrt(c-v).

p is proportional to v and m to $\mathrm{sqrt}(1+v^2)$. Thus:

H (a,b) is proportional to $\mathrm{sqrt}(1+v)\,(v, \mathrm{sqrt}(1+v^2)) + \mathrm{sqrt}(1-v)\,(\mathrm{sqrt}(1-v^2), -v)$

or H (a,b) is proportional to $\mathrm{sqrt}(1+v)\,(v, \mathrm{sqrt}(1+v^2)) + \mathrm{sqrt}(1-v) \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} (v, \mathrm{sqrt}(1+v^2))$

Again, the acceleration type matrix is appearing. It seems that H is inducing a kind of periodic bouncing motion with an acceleration matrix flipping from one state to the other.
It seems that (1,0), the positive velocity eigenvector is converted into forward and backward eigenvectors. The same occurs for the negative velocity eigenvector (0,1). The overall sqrt(1+v) and sqrt(1-v) weights ensure that there is an overall motion of v to the right. Thus, it almost seems as if the H operator is inducing a time dependent oscillatory motion into (1,0) and (0,1). The overall system moves forward with v, but there is internal back and forth motion, perhaps associated with an oscillation such as light.

Let us now consider specific zitterbewegung results for X(t), V(t) and dV(t)/dt from [9].

$X(t) = X(0) + tpH^{-1} + i\ .25/E\ dV/dt\ (\cos(2Et)-1) + .5/E \sin(2Et) \sin(a) (-V + \cos(a) H/E)$

The X(t) equation shows an oscillation along the direction of motion with an amplitude related to (-V + cos(a) H/E)

$V(t)= V + .5\ i \sin(2Et)\ dV/dt + \sin^2(Et) (\cos(a) H/E - V)$

dV(t)/dt proportional to dv/dt (1 - 2 sin2(Et) + i2sin(2Et) (-V + cos(a) H/E)

It can be seen that in all three cases, a term of (-V + cos(a) H/E) appears. This term is explicitly:

sin(a) | -sin(a) cos(a)| where v=cos(a). (4)
| cos(a)   sin(a)|

To get a sense of this operator, consider it operating on (1,0) the positive velocity vector. This yields:

{ (1,0) sqrt [$1-v^2$] - v (0,1) } sqrt($1-v^2$)   (5)

Again, we see the characteristic conversion of (1,0) into backward and forward pieces of (1,0) and (0,1) with weights fo the light triangle or weights of p/E and m/E. It is as if an oscillation were present driving this piece of the zitterbewegung. It is interesting to note that in this interpretation, there is time dependent oscillatory motion without any explicit time dependence appearing.

De Broglie Wavelength and Zitterbewegung

Zitterbewegung predicts frequencies based on exp(-iHt) namely cos(2Et). In special relativity, if one sets c=1, then p=Ev. Thus, momentum appears to be an energy flux through space. If energy E represents a physical frequency as zitterbewegung calculations predict, then momentum represents this ¨frequency dragged through space¨ or a wavelength. In fact, p=Ev means 1/v = E/p = frequency x wavelength. E was already identified as frequency (or related to frequency-there is an issue of a factor of 2) so 1/p is associated with the wavelength. De Broglie already made such specific suggestions in the 1920s.

Zitterbewegung calculations may give a physical picture of the wavelength as there are specific periodic motions in space. In the 2x2 model, zitterbewegung describes motion on a sphere if one interprets the periodic motion terms cos(2Et) etc as being projections of circular motion on an axis. It might be possible to try to assign a de Broglie wavelength to the region of space affected by the zitterbewegung motion. It appears that Prof. A. Niehaus does this in [13]. It seems, however, that one can also consider the fact that p/E appears in the weights of (5). Thus, the momentum is an important part of a type of internal back and forth motion.(It should be noted that the expectation values of the time-dependent zitterbewegung terms X(t), V(t) and dV(t)/dt all vanish when taken with the positive energy eigenvector.) Thus, there is a connection between the linear momentum p and internal periodic motion. Zitterbewegung already shows that there is a periodicity in time or frequency related to 2E. The internal motion related to p also occurs over a certain length, so it seems that the term (-V + cos(a) H) is linking p and ¨wavelength¨. Here, this amplitude is part X(t) in the direction of motion.

This may be important if the internal motion related to (-V +cos(a) H) requires a certain ¨wavelength¨ length in which it occurs. For a particle in free space, there is no issue, but once the particle is trapped in a box, then there could be problems if there is not enough space in which a ¨physical¨ periodic internal motion can occur. One may arguing that the particle must complete an integer number of internal motion cycles in order to be in resonance.If one calculates the dispersion sqrt{<E eigenvector| $(X-Xave)^2$ | E eigenvector>} with

|E eigenvector> = (cos(a/2), sin(a/2)) and uses the portion of X equal to (5) times 1/E one finds Xave=0 and Xrms= m/E*E where E=m/sin(a). This Xrms is not equal to 1/p, but does decrease with increasing p. It was already suggested that using p=Ev one can obtain the De Broglie relationship for p if E is taken as a frequency.

Conclusion

In conclusion, the 2x2 matrix model seems to be entirely based on the notion of light bouncing back and forth as described by the +E eigenvector (sqrt(1+v), sqrt(1-v)) which is proportional to (cos(a/2), sin(a/2)). The velocity matrix V appears to specifically pertain to this picture as well. It was also seen that there is an acceleration matrix dV/dt related to rotations of the vector (cos(a/2), sin(a/2)) which represent accelerations to new velocities. The velocity matrix dV/dt is proportional to [H,V] which is consistent with exp(-iHt) being an evolution operator just as in quantum mechanics. In earlier notes [2]-[7], it was argued in a loose fashion that exp(-iHt) was an evolution operator without explicitly noting that [H,V]=adV/dt (where a is a constant) can lead to this result. The operator exp(-iHt) is the ¨driver¨ of zitterbewegung calculations as it is the source of the ¨frequency¨ in these calculations. It seems, however, that it has deeper roots linked to a scenario of a photon bouncing back and forth. This appears to be the basis of the 2x2 matrix approach used in [2]-[7] and of the resulting velocity and acceleration matrices. It also appears that the zitterbewegung approach allows for a physical picture of frequency as related to energy. If momentum is considered as an energy flux, then this may describe momentum as being proportional to 1/wavelength. In addition, zitterbewegung calculations show physical periodic motion in space which may physically represent the ¨wavelength¨.

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