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Defuzzified the Cost Parameters and the Average Total Cost by Signed Distance Techniques using Trapezoidal Fuzzy Number

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Abstract : In this papers , a fuzzy production inventory model for constant deteriorating items is devolved . During production time, the rate of demand is liner stock dependent demand, once reached optimum quantity level , the demand rate is modified into non-linear price and liner stock dependent demand. Here , we have considered a price discount for the partially perishable item and shortages are not allowed In real life , we cannot define all cost parameters precisely due to imprecision or uncertainty , so we have defined the set up cost, holding cost, deteriorating cost ,production cost and discount price is assumed a trapezoidal fuzzy numbers and defuzzified the cost parameters and the average total cost by signed distance technique , centroid technique and graded mean integration technique . The numerical examples are given to the developed crisp and fuzzy models. A sensitivity analysis is additionally given to point out the effect of change of the parameters.

Keywords: Production inventory, nonlinear price and linear stock dependent demand, deterioration trapezoidal fuzzy numbers, defuzzification, signed distance technique centroid technique and graded mean integration technique.

SECTION-1 INTRODUCTION

In a manufacturing process the demand need not be same throughout the cycle. Hence we considered the following situations in our model. During the production time the demand is linear stock dependent demand once the production has reached the optimal level, we consider the demand is linear price and stock dependent demand. Many researchers think about the deteriorating items of their models but they have not taken into account the production losses due to manufacturing defects and machine faults. In this paper we have added the losses of the items in our model. Apart from that the price discount play an important role in the inventory problems, so we have included the price discount in the total average cost. Generally in business environment the cost parameters are changed with their original values . therefore, these parameters cannot be considered to be constant . so we applied the fuzzy concept to these cost parameters and which are defuzzified by signed distance technique, centroid technique and graded mean integration technique to obtain the optimal values of total average cost. Pervin.M et al [1] presented an inventory model with price-and stock-dependent demand. Halim.M.A [2] presented an inventory model for deteriorating items with nonlinear price and stock dependent demand. Palanivel.M et al[3] developed partial backlogging inventory model with price and stock level dependent demand. Other related model on inventory systems with stock-dependent consumption rate were developed by T.K.Datta et al[4].P.Dutta et al.,and Jaggi [5,6] have studied inventory model under fuzzy environment. Kumar &Rajput,[7,8] studied the model with time dependent. whereas, Mahata &De[9] invstigated the price dependent inventory. Authors [10-13] have discussed different types demand fuzzy inventory model with deteriorating

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item. . Sharmila.D, et al. [14] described Inventory model for deteriorating items involving fuzzy with shortages and exponential demand. Maragatham,M. et al [15] studied a fuzzy inventory model for deteriorating items with price dependent demand. Sanhita,B et al [16] used arithmetic operations on generalized trapezoidal fuzzy number and its applications SYed,J. et al [17] express a fuzzy inventory model without shortages using signed distance method.

Mohammad Abdul Halim[18],et al described an overtime production inventory model for deteriorating items with nonlinear price and stock dependent demand. This investigation was followed by several researchers like . A.A. Shaikh [19] Khan at al [20,21&22] derived production inventory model with partial trade credit policy and reliability. This paper has presented a production inventory model with constant rate of deterioration, where we considered various cost such as set up cost holding cost, price discount cost and cost of deterioration items are taken an trapezoidal fuzzy numbers. Later on, the fuzzy total cost and cycle time are defuzzified by using signed distance technique,centroid technique and graded mean integration technique.

II.ASSUMPTIONS AND NOTATIONS

A. Assumptons

Lead time is zero and shortages are not allowed.

Set up cost, holding cost, deteriorating cost, and rate of discount are assumed to be fuzzy.

Replenishment is instantaneous.

B. Notations

Demand rate: $D(q_1(t)) = a + q_1(t)$ is liner stock dependent demand during the period (t_1, T). where a and b are constants Demand rate: $D(q_1(t)) = \alpha p^{-\beta} - (a+q_2(t))$ is non-linear price and liner stock dependent demand during the period(t_1, T), where α and β are constants.

K = Production rate/ unit time.

ϕ = Deterioration fraction of product/ unit time

θ = Deterioration rate / unit time.

C_0 = Set up cost/ unit.

$\tilde{C}_0$ = Fuzzy set up cost/r unit

C_1 = Holding cost / unit.

$\tilde{C}_1$ = Fuzzy holding cost / unit.

C_2 = Deterioration cost/ unit.

$\tilde{C}_2$ = Fuzzy deterioration cost/ unit.

C_3 = Production cost / unit.

$\tilde{C}_3$ = Fuzzy production cost/ unit.

m = Rate of discount / unit.

$\tilde{m}$ = Fuzzy rate of discount/r unit.

T = Cycle length.

$\tilde{T}$ =Fuzzy cycle length

T_S = Defuzzified cycle length, using signed distance technique .

T_G = Defuzzified cycle length, using graded mean integration technique.

T_{Cen} =Defuzzified cycle length, using centroid technique.

TC = Total average cost.

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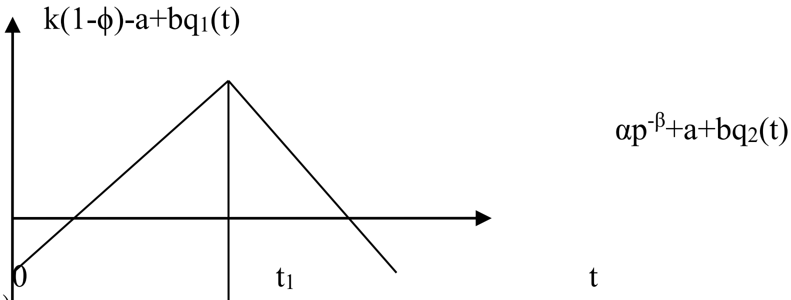
$\tilde{TC}$ = Fuzzified value of total average cost TC.
TC_S = Defuzzified value of fuzzy total average cost ,
using signed distance technique.
TC_G = Defuzzified value of fuzzy total average cost ,
using graded mean integration technique.
TC_{Cen} = Defuzzified value of fuzzy total average cost,
using centroid technique.

Here a production model is formulated, during production time $(0, t_1)$ the demand rates are stock dependent demand, after that demand rate is non-linear price and stock amount linearly dependent demand. We have also been taken into account the loss of production quantity due to faulty machine ,old machine and manufacturing defect etc., from the actual production quantities and also some production quantities deteriorates at the time of production.
At t=0 the production starts and production continues up to the time $t=t_1$.The stock amount, accumulated during the production period $t=t_1$, which is gradually diminishes to zero at the time t=T due to the meeting up of the non-linear price and stock amount linearly dependent demand.
The behavior of the production system at any time during a given cycle is presented in the Figure:1
The governing equation describing inventory q(t) with any time t is given



$$\frac{dq_1(t)}{dt}+\theta q_1(t)=k(1-\varphi)-(a+bq_1(t)),\ 0\le t\le t_1 \qquad (1)$$

$$\frac{dq_2(t)}{dt}+\theta q_2(t)=-\alpha p^{-\beta}-(a+bq_1(t)),\ t_1\le t\le T \qquad (2)$$

with boundary conditions $q_1(0) = 0$, $q_1(t_1) = q_2(t_1)$ and $q_2(T) =0$.
Solving these equations and using boundary conditions we have

$$q_1(t)=\frac{k(1-\varphi)-a}{\theta+b}\left(1-e^{-(\theta+b)t}\right) \qquad (3)$$

And $$q_2(t)=\frac{\alpha p^{-\beta}+a}{\theta+b}\left(e^{(\theta+b)(T-t)}-1\right) \qquad (4)$$

Now we find t_1 by using the boundary conditions $q_1(t_1) = q_2(t_1)$

$$\frac{k(1-\varphi)-a}{\theta+b}\left(1-e^{-(\theta+b)t_1}\right)=\frac{\alpha p^{-\beta}+a}{\theta+b}\left(e^{(\theta+b)(T-t_1)}-1\right) \qquad (5)$$

$$t_1=\frac{1}{2(\theta+b)R^2}\left[AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)-A^2(\theta+b)^2T^2\right] \qquad (6)$$

Where R = $k(1-\varphi)+\alpha p^{-\beta}$ k ; $A=a+\alpha p^{-\beta}$

Now various costs associated with the models are
Total Set up costs is SC=C_0, $0\le t\le T$ (7)

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Total production costs is PC= $c_3 k(1-\varphi)t_1$, $0 \le t \le t_1$ (8)

Total holding costs is HC, for the duration of $0 \le t \le T$ is

$$HC = C_1\left[\int_0^{t_1} q_1(t)dt + \int_{t_1}^{T} q_2(t)dt\right] \quad (9)$$

$$HC = C_1\left[\int_0^{t_1} \frac{k(1-\phi)-a}{\theta+b}\left(1-e^{-(\theta+b)t}\right)dt + \int_{t_1}^{T} \frac{\alpha p^{-\beta}+a}{\theta+b}\left(e^{(\theta+b)(T-t)}-1\right)dt\right] \quad (10)$$

$$HC = \frac{C_1}{\theta+b}\left[Rt_1 - AT\right] \quad (11)$$

Total deterioration costs for the duration of $0 \le t \le T$ is

$$DC = C_2\left[\int_0^{t_1} \theta q_1(t)dt + \int_{t_1}^{T} \theta q_2(t)dt\right]$$

$$DC = \frac{C_2\theta}{\theta+b}\left[Rt_1 - AT\right] \quad (12)$$

Total price discount is PD for the duration of $0 \le t \le T$ is

$$PD = C_3 m\left[\int_0^{t_1}\left(a + bq_1(t)\right)dt + \int_{t_1}^{T}\left(\alpha p^{-\beta} + \left(a + bq_2(t)\right)\right)dt\right] \quad (13)$$

$$PD = C_3 m\left[\frac{b}{2}Rt_1^2 - \left(\alpha p^{-\beta} + bTA\right)t_1 + AT + \frac{b}{2}AT^2\right] \quad (14)$$

The average total cost per unit time

$$\mathrm{TC} = \frac{1}{\mathrm{T}}\left[\mathrm{PC} + \mathrm{SC} + \mathrm{HC} + \mathrm{DC} + \mathrm{PD}\right] \quad (15)$$

Substituting the values of t1 by (6) in (15), we get

$$\mathrm{TC} = \frac{1}{\mathrm{T}}\left\{\begin{array}{l} C_3 k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\mathrm{AR}\left(2(\theta+b)\mathrm{T} + (\theta+b)^2\mathrm{T}^2\right) - \mathrm{A}^2(\theta+b)^2\mathrm{T}^2\right]\right) \\ + C_0 \\ + \left(\frac{C_1 + C_2\theta}{\theta+b}\right)\left(\frac{1}{2(\theta+b)R}\left[\mathrm{AR}\left(2(\theta+b)\mathrm{T} + (\theta+b)^2\mathrm{T}^2\right) - \mathrm{A}^2(\theta+b)^2\mathrm{T}^2\right] - \mathrm{AT}\right) \\ + C_3 m\left[\begin{array}{l} -\frac{b\mathrm{A}^2\mathrm{T}^2}{2R} - \alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\mathrm{AR}\left(2(\theta+b)\mathrm{T} + (\theta+b)^2\mathrm{T}^2\right) - \mathrm{A}^2(\theta+b)^2\mathrm{T}^2\right]\right) \\ + \mathrm{AT} + \frac{b\mathrm{AT}^2}{2} \end{array}\right] \end{array}\right\} \quad (16)$$

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$$TC=\left\{\begin{array}{l} C_3k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(2(\theta+b)+(\theta+b)^2T\right)\\ -A^2(\theta+b)^2T\end{array}\right]\right)+\frac{C_0}{T}\\ +\left(\frac{C_1+C_2\theta}{\theta+b}\right)\left(\frac{1}{2(\theta+b)R}\left[\begin{array}{l}AR\left(2(\theta+b)+(\theta+b)^2T\right)\\ -A^2(\theta+b)^2T\end{array}\right]-A\right)\\ +C_3m\left[\begin{array}{l}-\frac{bA^2T}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(2(\theta+b)+(\theta+b)^2T\right)\\ -A^2(\theta+b)^2T\end{array}\right]\right)\\ +A+\frac{bAT}{2}\end{array}\right]\end{array}\right\} \quad (17)$$

Now the optimum value of T, which minimize the total average cost TC , the value of T for which $\frac{\partial TC}{\partial T}=0$ gives,

$$T=\sqrt{\frac{C_0 2R/\left(AR-A^2\right)}{\left(\begin{array}{l}C_3R^2K(1-\phi)(\theta+b)\\ +R(C_1+C_2\theta)+C_3m\left(bR-\alpha P^{-\beta}\right)\\ (\theta+b)\end{array}\right)}}$$

III FUZZY THE MODEL AND SOLUTION PROCEDURE

Next we the fuzzfy the parameters C_0, C_1, C_2, C_3, C_4 and m. Let

$$\tilde{C}_0=(w_0,x_0,y_0,z_0),\ \tilde{C}_1=(w_1,x_1,y_1,z_1),\ \tilde{C}_2=(w_2,x_2,y_2,z_2),\ \tilde{C}_3=(w_3,x_3,y_3,z_3)$$

$$\tilde{m}=(m_1,m_2,m_3,m_4),\ \tilde{C}_3m=(w_3m_1,x_3m_2,y_3m_3,z_3m_4)$$

Then $T\tilde{C}=\frac{1}{T}\left(T\tilde{C}_1,T\tilde{C}_2,T\tilde{C}_3,T\tilde{C}_4\right)$ (say) ,Where

$$T\tilde{C}_1=\frac{1}{T}\left\{\begin{array}{l} w_3k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)\\ -A^2(\theta+b)^2T^2\end{array}\right]\right)+w_0\\ +\left(\frac{w_1+w_2\theta}{\theta+b}\right)\left(\frac{1}{2(\theta+b)R}\left[\begin{array}{l}AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)\\ -A^2(\theta+b)^2T^2\end{array}\right]-AT\right)\\ +w_3m_1\left[\begin{array}{l}-\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(\begin{array}{l}2(\theta+b)T+\\ (\theta+b)^2T^2\end{array}\right)\\ -A^2(\theta+b)^2T^2\end{array}\right]\right)\\ +AT+\frac{bAT^2}{2}\end{array}\right]\end{array}\right\}$$

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$$\tilde{TC}_2 = \frac{1}{T}\left\{\begin{array}{l} x_3 k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l} AR\left(2(\theta+b)T+(\theta+b)^2T^2\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]\right)+x_0 \\ +\left(\frac{x_1+x_2\theta}{\theta+b}\right)\left(\frac{1}{2(\theta+b)R}\left[\begin{array}{l} AR\left(2(\theta+b)T+(\theta+b)^2T^2\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]-AT\right) \\ +x_3 m_2\left[\begin{array}{l} -\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]\right) \\ +AT+\frac{bAT^2}{2} \end{array}\right] \end{array}\right\}$$

$$\tilde{TC}_3 = \frac{1}{T}\left\{\begin{array}{l} y_3 k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]\right)+y_0 \\ +\left(\frac{y_1+y_2\theta}{\theta+b}\right)\left(\frac{1}{2(\theta+b)R}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]-AT\right) \\ +y_3 m_3\left[\begin{array}{l} -\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]\right) \\ +AT+\frac{bAT^2}{2} \end{array}\right] \end{array}\right\}$$

$$\tilde{TC}_4 = \frac{1}{T}\left\{\begin{array}{l} z_3 k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]\right)+z_0 \\ +\left(\frac{z_1+z_2\theta}{\theta+b}\right)\left(\frac{1}{2(\theta+b)R}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]-AT\right) \\ +z_3 m_4\left[\begin{array}{l} -\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l} AR\left(\begin{array}{l} 2(\theta+b)T \\ +(\theta+b)^2T^2 \end{array}\right) \\ -A^2(\theta+b)^2T^2 \end{array}\right]\right) \\ +AT+\frac{bAT^2}{2} \end{array}\right] \end{array}\right\}$$

By sing signed distance technique , defuzzified the total average cost $\tilde{TC}_S$ is given by

$$\tilde{TC}_S = \frac{1}{4}\left(\tilde{TC}_1+\tilde{TC}_2+\tilde{TC}_3+\tilde{TC}_4\right)$$

$$TC_S = \frac{1}{4T}\left\{\begin{array}{l}(w_3+x_3+y_3+z_3)k(1-\varphi)\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(\begin{array}{l}2(\theta+b)T\\+(\theta+b)^2T^2\end{array}\right)\\-A^2(\theta+b)^2T^2\end{array}\right]\right)\\+(w_0+x_0+y_0+z_0)\\+\left(\frac{(w_1+x_1+y_1+z_1)+(w_2+x_2+y_2+z_2)\theta}{\theta+b}\right)\\\left(\frac{1}{2(\theta+b)R}\left[AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)-A^2(\theta+b)^2T^2\right]-AT\right)\\+(w_3m_1+x_3m_2+y_3m_3+z_3m_4)\\\left[\begin{array}{l}-\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)\\-A^2(\theta+b)^2T^2\end{array}\right]\right)\\+AT+\frac{bAT^2}{2}\end{array}\right]\end{array}\right\}$$

Now $\frac{d(TC_S)}{dT}=0,$ given

$$T_S = \sqrt{\frac{2R^2(w_0+x_0+y_0+z_0)}{(AR-A^2)\left[\begin{array}{l}(w_3+x_3+y_3+z_3)R^2k(1-\varphi)(\theta+b)\\+R\left[\begin{array}{l}(w_1+x_1+y_1+z_1)+\\(w_2+x_2+y_2+z_2)\theta\end{array}\right]\\+(w_3m_1+x_3m_2+y_3m_3+z_3m_4)\\(bR-\alpha p^{-\beta}(\theta+b))\end{array}\right]}}$$

By using graded mean integration technique, defuzzified the total average cost is given by

$$TC_G = \frac{1}{6}\left(T\tilde{C}_1+2T\tilde{C}_2+2T\tilde{C}_3+T\tilde{C}_4\right)$$

$$TC_G = \frac{1}{6T}\left\{\begin{array}{l}(w_3+2x_3+2y_3+z_3)k(1-\varphi)\\\left(\frac{1}{2(\theta+b)R^2}\left[AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)-A^2(\theta+b)^2T^2\right]\right)\\+(w_0+2x_0+2y_0+z_0)\\+\left(\frac{(w_1+2x_1+2y_1+z_1)+(w_2+2x_2+2y_2+z_2)\theta}{\theta+b}\right)\\\left(\frac{1}{2(\theta+b)R}\left[AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)-A^2(\theta+b)^2T^2\right]-AT\right)\\+(w_3m_1+x_3m_2+y_3m_3+z_3m_4)\\\left[\begin{array}{l}-\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2(\theta+b)R^2}\left[\begin{array}{l}AR\left(2(\theta+b)T+(\theta+b)^2T^2\right)\\-A^2(\theta+b)^2T^2\end{array}\right]\right)\\+AT+\frac{bAT^2}{2}\end{array}\right]\end{array}\right\}$$

Now $\frac{d(TC_G)}{dT}=0,$ given

$$T_G = \sqrt{\frac{2R^2\left(w_0+2x_0+2y_0+z_0\right)}{\left(AR-A^2\right)\begin{bmatrix}\left(w_3+2x_3+2y_3+z_3\right)R^2k\left(1-\varphi\right)\left(\theta+b\right)\\ +R\left[\left(w_1+2x_1+2y_1+z_1\right)+\left(w_2+2x_2+2y_2+z_2\right)\theta\right]\\ +\left(w_3m_1+x_3m_2+y_3m_3+z_3m_4\right)\left(bR-\alpha p^{-\beta}\left(\theta+b\right)\right)\end{bmatrix}}}$$

By using centroid technique, defuzzified the total average cost TC_{Cen} is

$$TC_{Cen} = \frac{1}{3}\left(T\tilde{C}_1+T\tilde{C}_2+T\tilde{C}_3+T\tilde{C}_4\right)$$

$$TC_{Cen} = \frac{1}{3T}\left\{\begin{matrix} CC_3k\left(1-\varphi\right)\left(\frac{1}{2\left(\theta+b\right)R^2}\begin{bmatrix}AR\begin{pmatrix}2\left(\theta+b\right)T\\ +\left(\theta+b\right)^2T^2\end{pmatrix}\\ -A^2\left(\theta+b\right)^2T^2\end{bmatrix}\right)+CC_0\\ +\left(\frac{CC_1+CC_2\theta}{\theta+b}\right)\left(\frac{1}{2\left(\theta+b\right)R}\begin{bmatrix}AR\begin{pmatrix}2\left(\theta+b\right)T\\ +\left(\theta+b\right)^2T^2\end{pmatrix}\\ -A^2\left(\theta+b\right)^2T^2\end{bmatrix}-AT\right)\\ +CC_3m\begin{bmatrix}-\frac{bA^2T^2}{2R}-\alpha p^{-\beta}\left(\frac{1}{2\left(\theta+b\right)R^2}\begin{bmatrix}AR\begin{pmatrix}2\left(\theta+b\right)T\\ +\left(\theta+b\right)^2T^2\end{pmatrix}\\ -A^2\left(\theta+b\right)^2T^2\end{bmatrix}\right)\\ +AT+\frac{bAT^2}{2}\end{bmatrix}\end{matrix}\right\}$$

Now $\frac{d(TC_{Cen})}{dT}=0$, given

$$T_{Cen} = \sqrt{\frac{2R^2CC_0}{\left(AR-A^2\right)\begin{bmatrix}CC_3R^2k\left(1-\varphi\right)\left(\theta+b\right)\\ +R\left[CC_1+CC_2\theta\right]+CC_3m\left(bR-\alpha p^{-\beta}\left(\theta+b\right)\right)\end{bmatrix}}}$$

where

$$CC_0 = \frac{1}{3}\left(\frac{y_0^2+z_0^2+\left(y_0z_0\right)-w_0^2-x_0^2-\left(w_0x_0\right)}{y_0+z_0-w_0-x_0}\right)$$

$$CC_1 = \frac{1}{3}\left(\frac{y_1^2+z_1^2+\left(y_1z_1\right)-w_1^2-x_1^2-\left(w_1x_1\right)}{y_1+z_1-w_1-x_1}\right)$$

$$CC_2 = \frac{1}{3}\left(\frac{y_2^2+z_2^2+\left(y_2z_2\right)-w_2^2-x_2^2-\left(w_2x_2\right)}{y_2+z_2-w_2-x_2}\right)$$

$$CC_3 = \frac{1}{3}\left(\frac{y_3^2+z_3^2+\left(y_3z_3\right)-w_3^2-x_3^2-\left(w_3x_3\right)}{y_3+z_3-w_3-x_3}\right)$$

$$CC_3m = \frac{1}{3}\left(\frac{\begin{matrix}\left(y_3m_3\right)^2+\left(z_3m_4\right)^2+\left(\left(y_3m_3\right)\left(z_3m_4\right)\right)\\ -\left(w_3m_1\right)^2-\left(x_3m_2\right)^2-\left(\left(w_3m_1\right)\left(x_3m_2\right)\right)\end{matrix}}{y_3m_3+z_3m_4-w_3m_1-x_3m_2}\right)$$

To illustrate the models with the following input data.

k=10, p=10, β=0.4, α=4, θ= 0.001, a = 5, b = 0.009 p = 12, $C_2=6$, ϕ=0.09

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$C_0 = 20,\ C_1 = 5,\ C_2 = 6,\ C_3 = 10,\ m = 0.01$

$\tilde{C}_0 = (w_0, x_0, y_0, z_0), \tilde{C}_1 = (w_1, x_1, y_1, z_1)\ \tilde{C}_2 = (w_2, x_2, y_2, z_2),\ \tilde{C}_3 = (w_3, x_3, y_3, z_3)$

$\tilde{m} = (m_1, m_2, m_3, m_4)\ \tilde{C}_3 m = (w_3 m_1, x_3 m_2, y_3 m_3, z_3 m_4)$

Table:1

Comparison between Crisp and Fuzzy models

Techniques	TC	T
Crisp	75.87651	0.9708770
Signed distance	86.46392	0.8680647
Grad mean integration	86.50317	0.8703597
Centroid	86.44400	0.8667861

- Sensitive Analysis

Based on the above values, we have examined the sensitivity analysis by changing parameters

Table -2 sensitivity analysis on K

	10	11	12	13	14
TC	82.37174	85.55445	88.17314	90.89257	93.49177
T	1.039942	0.922934	0.834904	0.761330	0.700303
TC_G	82.41281	85.59607	88.21506	90.93488	93.53442
T_G	1.039940	0.922932	0.834903	0.761329	0.700302
TC_S	82.42775	85.61120	88.23031	90.95027	93.54992
T_S	1.033994	0.922932	0.834902	0.761329	0.700302
TC_{Cen}	82.43522	85.61877	88.23793	90.95796	93.55768
T_{cen}	1.033994	0.922932	0.834902	0.761329	0.700302

Table -3 sensitivity analysis on θ

	0.001	0.002	0.003	0.004	0.005
TC	88.17314	88.82559	89.47016	90.10633	90.73386
T	0.834904	0.805879	0.779686	0.755890	0.734149
TC_G	88.21506	88.08675	89.51207	90.14824	90.77577
T_G	0.834903	0.805882	0.779685	0.755889	0.734148
TC_S	88.23031	88.86751	89.52732	90.16348	90.79101
T_S	0.834902	0.805878	0.779684	0.755889	0.734148
TC_{Cen}	88.23793	88.89038	89.53494	90.17111	90.79863
T_{cen}	0.834902	0.805878	0.779682	0.755889	0.734147

Table -4 sensitivity analysis on C_0

Defuzzified values of C_0	24	22	20	18	16
TC_G	93.00603	90.61055	88.21506	85.81958	83.42409
T_G	0.914590	0.875653	0.834903	0.792058	0.746760
TC_S	93.02128	90.62579	81.23031	85.83482	83.43934
T_S	0.914590	0.875653	0.834902	0.792058	0.746759
TC_{Cen}	93.02890	90.63348	88.23793	85.84244	83.44696
T_{cen}	0.914589	0.875653	0.834902	0.792058	0.746759

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Table -5 sensitivity analysis on C_1

efuzzified values of C_1	7	6	5	4	3
TC_G	89.50786	89.50656	88.21506	86.92356	85.63206
T_G	0.813246	0.813267	0.834903	0.858362	0.833917
TC_S	89.52310	89.52181	88.23031	86.93880	85.64730
T_S	0.813246	0.813267	0.834902	0.858362	0.883916
TC_{Cen}	89.53072	89.52943	88.23793	86.92356	85.65492
T_{cen}	0.813246	0.813266	0.834902	0.858361	0.883916

Table -6 sensitivity analysis on C_2

Defuzzified values of C_2	8	7	6	5	4
TC_G	88.21769	88.21635	88.21506	88.21377	88.21248
T_G	0.834858	0.834880	0.834903	0.834925	0.834948
TC_S	88.23289	88.23160	88.23031	88.22901	88.22772
T_S	0.834857	0.834880	0.834902	0.834925	0.834947
TC_{Cen}	88.24051	88.23922	88.23793	88.23664	88.23535
T_{cen}	0.834857	0.834879	0.834902	0.834925	0.834947

Table -7 sensitivity analysis on C_3

Defuzzified valuesof C_3	8	9	10	11	12
TC_G	76.66446	82.43976	88.21506	93.99036	99.76567
T_G	0.903467	0.867158	0.834903	0.805998	0.779901
TC_S	76.67970	82.45500	88.23031	93.00561	99.78091
T_S	0.903466	0.867158	0.834902	0.805997	0.779901
TC_{Cen}	76.68732	82.46263	88.23793	93.01323	99.78853
T_{cen}	0.93466	0.867158	0.834902	0.805997	0.779901

Table -8 sensitivity analysis on m

Defuzzified values of m	0.006	0.008	0.010	0.012	0.014
TC_G	87.98639	88.10072	88.21506	88.32940	88.44374
T_G	0.834910	0.834906	0.834903	0.834899	0.834896
TC_S	87.00163	88.11597	88.23031	88.34464	88.45898
T_S	0.834909	0.834906	0.834902	0.834899	0.834895
TC_{Cen}	88.00926	88.12359	88.23793	88.35227	88.46660
T_{cen}	0.834909	0.834906	0.834902	0.834898	0.834895

IV OBSERVATION

The following interesting observations are noted on the basis of the rate sensitivity analysis.

1) From table -1 and table -2 as we raise the production and deterioration cost the optimum cycle length of T, TG, TS and TCen decrease in contrast the optimum values of total average cost TC, TCG, TCS and TCCen increase.

2) From table -3, table -4 and table -5 as we decrease the set up cost and holding cost, we noted that the total average cost for all models decrease and the optimum cycle length of T, TG, TS and TCen also decreases.
3) From the table -6 and table -7 as we increased the production cost and rate of discount, we observed that optimum cycle length TG, TS and TCen decreases and the total average cost (for all three models) increases.
4)The comparison of optimum values found in crisp and fuzzy models, the total average cost of the fuzzy models more than the crisp model but less sensitive in optimal cycle length in crisp and fuzzy models.

V. CONCLUSION: In this paper ,we have presented production inventory model for deteriorating items under fuzzy environment, naturally production inventory model have consists the set up cost, holding cost, production cost and deterioration cost in addition that they have considered the price discount for our model . Here we have used the linear stock dependent demand during the production time and once we reach the optimum quantity level we considered the demand rate in non-linear price and linear stock dependent. In fuzzy environment all related inventory values are trapezoidal fuzzy numbers the optimum values of total average cost and cycle length for fuzzy models are defuzzifed into signed distance, centroid and graded mean integration technique.

We observed that, in the fuzzy model, the total average cost is minimum with corresponding cycle length T, when graded mean integration technique is used. On the other hand the cycle length time is minimum with corresponding total average cost when centroid technique of defuzzification is used.

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