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Analysed Temporal Yager's Pythagorean Fuzzy Collections on R-Ideal Theory

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Abstract: The notion of temporal Pythagorean fuzzy subgroups (TPFS) is introduced, and related properties are investigated. Characterizations of a temporal Pythagorean fuzzy subgroup (TPFG) are established, and how the images or inverse images of temporal Pythagorean fuzzy subgroups become temporal Pythagorean fuzzy subgroups is studied. Ideal concepts are discussed in many mathematical applications. Various author has been studied and analytical in different ways. In this article, the idea of bipolar temporal Pythagorean uncertainty sub algebra's in terms of R-ideals is planned. Also the correlation among bipolar temporal Pythagorean uncertainty soft ideal and bipolar temporal Pythagorean uncertainty soft R-ideals is expressed some interesting ideas also analyzed.

Keywords: Fuzzy set, Interval valued fuzzy set, temporal fermatean fuzzy set, temporal Pythagorean fuzzy subgroup, level set, pre-image homomorphism, , Algebra, R-ideals

1.INTRODUCTION: later the idea of uncertainty collections of Zadeh [21], lee [10] presented another trend of uncertainty collections called bipolar valued uncertainty sets (BVUS). bipolar valued uncertainty set defined over the interval [-1,1] which was to be extended from the ordinary fuzzy set interval [0, 1]. the idea of bipolar parameterized collections and several identification of bipolar parameterized collection were presented by Shabir and Naz [16]. Abdulla et al. [1] studied the idea of bipolar uncertainty parameterized collections by combining parameterized collections and bipolar uncertainty collections sponsored by Zhang [19, 20], and given parametrical ideal identifications of bipolar uncertainty parameterized collections. Akram et al. [3] explained an idea of positive and negative uncertainty soft sub semi group and positive and negative uncertainty soft-ideals in a semi group. the minus membership function and the plus membership function defined in [-1, 0] and [0, 1] in bipolar uncertainty setting. in this bipolar uncertainty setting '0' refers that the elements are subjected to irrelevant. they are familiar representation and down word representation. the familiar forms of positive and negative uncertainty collections are used in their representations. in 2011, positive and negative fuzzy k−sub algebras are analyzed by Farhatnisar [5]. stimulated by the notions in recent times, the result of bipolar valued fuzzy sub algebras/ideals of a bf-algebra [4] has discussed by applying the notion of bipolar valued uncertainty collection (BVUS) in bf-algebras [4]. fermatean uncertainty bipolar model as a combination of uncertainty bipolar model and pythagorean uncertainty bipolar. group symmetry analyzes a moral character to molecule structures. the author [18] coined the fermatean uncertainty set (FUS) with its relational measures. collections data between parameterized collections were studied by Maji et al. [12]. some author [2] explained various identifications on the parameterized collections, and Sezgin and Atagun [17] investigated on parameterized set identifications as well. in this view, we analyze various domain of ideals and investigate some temporal fermatean uncertainty collections and its properties. In this article, the idea of bipolar temporal Pythagorean uncertainty sub algebra's in terms of R-ideals is planned. Also the correlation among bipolar temporal Pythagorean uncertainty soft ideal and bipolar temporal Pythagorean uncertainty soft R-ideals is expressed some interesting ideas also analyzed.

2.Prelimineries:

Let I be a closed Interval, i.e., I = [0,1]. By an Interval number we mean a closed sub interval $\bar{a}$ = $[\bar{a}, a^+]$ of I, where $0 \le \bar{a} \le a^+ \le 1$, denoted by D[0,1] the set of interval l numbers. Let us define what is known as refined minimum of two elements in D[0,1]. We also define the symbols " ≤ " , " ≥ " , " = " in case of two elements in D[0,1].

Consider two Interval numbers $\bar{a}_1 = [\bar{a}_1, a_1^+]$ and $\bar{a}_2 = [\bar{a}_2, a_2^+]$, then

$$\min\{\bar{a}_1, \bar{a}_2\} = \{\min\{\bar{a}_1, \bar{a}_2\}, \min\{a_1^+, a_2^+\}\},$$

$\bar{a}_1 \ge \bar{a}_2$ if and only if $\bar{a}_1 \ge \bar{a}_2$ and $a_1^+ \ge a_2^+$ and Similarly we may have $\bar{a}_1 \le \bar{a}_2$ and $\bar{a}_1 = \bar{a}_2$

Let $\bar{a}_i$ ε D[0,1] when i ε Λ . We define

΄Y{ $\inf_{i \varepsilon \Lambda}$ } $\bar{a}_I$ = [{ $\inf_{i \varepsilon \Lambda} \bar{a}_i$} , { $\inf_{i \varepsilon \Lambda}$ a_i^+}] and

΄Y{ $\sup_{i \varepsilon \Lambda}$ } $\bar{a}_I$ = [{ $\sup_{i \varepsilon \Lambda} \bar{a}_i$} , { $\sup_{i \varepsilon \Lambda}$ a_i^+}] .

An interval valued fuzzy set (IVFS) δ^{-}_A defined on a non empty set X is given by δ^{-}_A = { (x , δ^{-}_A (x) , δ^{+}_A) / x ε X }, which is briefly denoted by δ^{-}_A = [δ^{-}_A , δ^{+}_A } where δ^{-}_A and δ^{+}_A are two fuzzy sets in X such that $\delta^{-}_A (x) \leq \delta^{+}_A (x)$ for all x ε X.

δ^{-}_A (x) are called the degree of membership of an element x to δ^{-}_A , in which δ^{-}_A (x) and δ^{+}_A (x) are referred to as the lower and upper degrees respectively of membership of x to δ^{-}_A .

Definition 2.1 : Let E be the Universe and T be a non-empty set of time moments. Then a temporal fuzzy set (TFS) is an object having the form

A(T) = { (x, δ_A (x, t) / (x, t) ε E x T } here

(i) A C E is a fixed set.

(ii) δ_A (x, t) denote the degree of membership of the element (x, t) such that $0 \leq \delta_A (x, t) \leq 1$ for all (x, t) ε E X T.

Example 2.2 : Let X = [x_1. x_2. x_3 } be the set and T be the non – empty set of time moments.

t → x ↓	t_1	t_2
x_1	0.4	0.4
x_2	1.0	0.2
x_3	0.0	0.1

For brevity we write A instead of A(T) , the hesitation degree of a TFs is defined as $\pi_A (x, t) = 1$ **--** δ_A (x, t). Obviously, every ordinary Fuzzy set can be regarded as TFs for which T is a singleton set. All operations and operators can be defined for TFs in Atanassov. K.T. [1991].

Definitions 2.3 : Let A (T ‘) = { (x, δ_A (x, t) / E X T ‘ }

A (T ‘’) = { (x, δ_A (x, t) / E X T ‘’ } where T ‘ and T’’ have finite number of distinct time – elements or they are time interval. Then

A (T ‘) ⋂ B (T’’) = { (x, min { δ^{-}_A (x, t) , δ^{-}_B (x, t) }

A (T ‘) ⋃ B (T’’) = { (x, max { δ^{-}_A (x, t), δ^{-}_B (x, t) }

Also from definition of subset in Fuzzy fit theory subsets of TFs can be defined as the following:

A(T') $\subseteq$ B (T '') $\Leftrightarrow$ δ^{-}_{A} (x, t) , δ^{-}_{B} (x,t) for every (x, t) ε (T ' $\cup$ T''),

Where $\delta^{-}_{A}(x,t) = \begin{cases} \delta_A(x,t) & \text{if } t \, \varepsilon \, T'' \\ 0 & \text{if } t \, \varepsilon \, T'' - T' \end{cases}$ and

$\delta^{-}_{B}(x,t) = \begin{cases} \delta_B(x,t) & \text{if } t \, \varepsilon \, T'' \\ 0 & \text{if } t \, \varepsilon \, T' - T'' \end{cases}$

It is obviously seen that if T ' = T ''. δ^{-}_{A} (x, t) = { δ_A (x, t) , δ^{-}_{B} (x, t) = δ_B (x, t).

Let J be an arbitrary index set, then we define that T = $\bigcup i\varepsilon J$ Ti where Ti is a time set for each j ε J. Thus we extend the definition of Union and interjection of TFs family F = { Ai(Ti))= { (x, δ_{Ai} (x, t) / x ε E x Ti , i ε J }

As follows ($\bigcup i\varepsilon J$)A (Ti) = { (x, (max)$_{j \in J}$(δ^{-}_{A} (x, t) / (x, t) € E x T }

($\bigcap j \in J$) A(Ti) = { (x, (min)$_{j \in J}$(δ^{-}_{A} (x, t) / (x, t) € E x T }

Where (δ^{-}_{A} (x, t) = δ_A (x, t) if t € Tj

= 0 if t € T – Tj

Definition 2.4: [L.A. Zadeh, 1965] Let 'X' be a collection of all elements. An uncertainty collection 'A' falls from X is expressed as $A = \{(x : \mu A(x)) / x \in X\}$, where $\mu A : A \to [0,1]$ is the grade mapping of the uncertainty collections A.

Definition 2.5: [K. Lee, 2009] Let 'X' is a Universe. Then a bipolar uncertainty collection A on X is represented by plus membership map μA^{+}, that is, $\mu A^{+} : X \to [0, 1]$ and a negative membership map μA^{-}, (i, e), $\mu A^{-} : X \to [-1, 0]$. For the state of easy way, we always utilize the symbol $A = \{(x, \mu A^{+}, \mu A^{-}) / x \in X\}$.

In what follows let X denote the group under otherwise specified.

Definition 2.6: Let ' X ' be a non empty set. A temporal fermatean fuzzy set (TFFS) A in a set X is a structure A = { (x, δ^{-}_{A} (x ,t) , λ (x, t) / (x, t) € X x T } where is briefly denoted by A = < δ^{-}_{A} , λ > where δ^{-}_{A} = [δ^{-}_{A} , δ^{+}_{A}] is an temporal interval Fuzzy set (TIVFS) in X and λ is a fuzzy set (FS) in X. Denote C (X , T) the family of temporal fermatean fuzzy sets in X.

Definition 2.7: A TPFS A = < δ^{-}_{A} , λ > in X is called a temporal Pythagorean fuzzy sub group (TPFSG) of X if it satisfies

(i) $\delta^{-}_{A}(xy, t) \geq Ύ \min \{ \delta^{-}_{A}(x, t), \delta^{-}_{A}(y, t) \}$ and $\delta^{-}_{A}(x^{-1}, t) \geq \delta^{-}_{A}(x, t)\}$

(ii) $\lambda(xy, t) \leq \max\{ \lambda(x, t), \lambda(y, t)\}$ and $\lambda(x^{-1}, t) \leq \lambda(x, t)$, for all x,y € X and t € T.

Example 2.8: Let X be the Klein's four group. We have X = { e, a, b, ab } where $a^2 = e = b^2$ and ab = ba . We define $\delta^{-}_{A} = [\delta^{-}_{A}, \delta^{+}_{A}]$ and λ by

$$\delta^{-}_{A} = \begin{pmatrix} e & a & b & ab \\ [0.3,0.4] & [0.3,0.8] & [0.7,0.4] & [0.9,0.3] \end{pmatrix}$$

and λ =

$$\begin{pmatrix} e & a & b & ab \\ 0.1 & 0.7 & 0.9 & 0.3 \end{pmatrix}$$

Then A = (δ^{-}_{A} , λ) is a TPFSG of X.

Note 2.9: For any x, y € X and $\delta^{-}_{A}(y, t) > \delta^{-}_{A}(x, t)$ and $\lambda(y, t) < \lambda(x, t)$, then the equalities $\delta^{-}_{A}(xy, t) = \delta^{-}_{A}(x, t) = \delta^{-}_{A}(yx, t)$ and $\lambda(xy, t) = \lambda(x, t) = \lambda(yx, t)$ are not true.

Example 2.10:In the Klein's four group X = { e,a,b,ab), we define $\delta^{-}_{A} = [\delta^{-}_{A}, \delta^{+}_{A}]$ and λ by

$$\delta^{-}_{A} = \begin{pmatrix} e & a & b & ab \\ [0.3,0.9] & [0.1,0.7] & [0.1,0.9] & [0.3,0.7] \end{pmatrix}$$

$$\text{and } \lambda = \begin{pmatrix} e & a & b & ab \\ 0.2 & 0.6 & 0.4 & 0.6 \end{pmatrix}$$

Then A = (δ^{-}_{A} , λ) is a TPFSG of X.

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Note that δ⁻A (b, t) = [0.,0.9] > [0.1, 0.7]= δ⁻A (a,t) and
λ (b,t)= 0.4 < 0.6 = λ (a, t). But δ⁻A (ab) = [0.3,0.7] ≠ [0.1, 0.7]= δ⁻A (a, t).

Definition 2.11: [Senapati and Yager, 2019] Let 'X' is Universe of discovers. A fermatean uncertainty set (FUS) F in X is a domain has the formulation as $F=\{(x, m_F(x), n_F(x))/x \in X\}$, where $m_F(x): X \to [0, 1]$ *and* $n_F(x): X \to [-1, 0]$, which includes the result $0 \le (m_F(x))^2 + (n_F(x))^2 \le 1, \forall x \in X$, the numbers $m_F(x)$ denotes the power of elements and $n_F(x)$ denotes the power of non-membership of the element $x \in F$, all Pythagorean uncertainty set 'F' and $x \in F$. $\prod F(x) = \sqrt{1-(m_F(x))^2-(n_F(x))^2}$ is defined as the degree of middle of x to F. For convince, Senapati and Yager called $(m_F(x), n_F(x))$ Pythagorean fuzzy number (PFN) denoted by $F=(m_F, n_F)$.

Definition 2.12: [Moldtsov 1999] Let U is an initial Universe, P(U) is the power set of U and E is collection of all notations and A⊆E. A parameterized collections (δ_A, E) on the Universe 'U' is explained by the collections of order pairs $(\delta_A, E) = \{(e, \delta_A(e)) : e \in E, \delta_A \in P(U)\}$, where $\delta_A : E \to P(U)$ such that $\delta_A(e)=\phi$ *if* $e \notin A$. Here δ_A is known as tentative mapping of the parameterized collections.

Example 2.13: Let $U=\{v_1, v_2, v_3, v_4\}$ be a collection of four pants and $E = \{white(e_1), red(e_2), blue(e_3)\}$ be a collection of objects. If $A = \{e_1, e_2\} \subseteq E$. Let $\delta_A(e_1) = \{v_1, v_2, v_3, v_4\}$ and $\delta_A(e_2) = \{v_1, v_2, v_3\}$then we form the parameterized set $(\delta_A, E) = \{(e_1, \{v_1, v_2, v_3, v_4\}), (e_2, \{v_1, v_2, v_3\})\}$ over 'U' which symbolized the "color of the pants" which Mr. A is going to buy. This can be represented the soft set in the given format.

$\sum$	e_1	e_2	e_3
v_1	1	1	0
v_2	1	1	0
v_3	1	1	0
v_4	1	0	0

Definition 2.14: [Bipolar Pythagorean uncertainty soft set] Let 'X' is a collection of all elements. A bipolar Pythagorean uncertainty soft set (BPFUSS). $F=\left\{\left(u,m_F{}^P,n_F{}^P,m_F{}^N,n_F{}^N/u\in X\right)\right\}$, Where $m_F{}^P:X\to[0,1]$, $n_F{}^P:X\to[0,1]$, $m_F{}^N:X\to[0,1]$, $n_F{}^N:X\to[0,1]$ that are the mappings such that $0\le(m_F)^3+(n_F)^3\le1$ and $-1\le(m_F)^3+(n_F)^3\le0$ *and* $m_F{}^P(u)$ denotes positive membership degree, $n_F{}^P(u)$ represents positive non-membership degree, $n_F{}^N(u)$represents negative membership degree, $n_F{}^N(u)$ represents negative non-membership degree. The degree of indeterminacy.

$$\prod F^P(u)=\sqrt[3]{1-\left(m_F{}^P(u)\right)^3-\left(n_F{}^P(u)\right)^3}\ \ and\ \ \prod F^N(u)=\sqrt[3]{1-\left(m_F{}^N(u)\right)^3-\left(n_F{}^N(u)\right)^3}.$$

Definition2.15: Let $F_1=\left\{\left(u,m_{F_1}{}^P,n_{F_1}{}^P,m_{F_1}{}^N,m_{F_1}{}^N/u\in X\right)\right\}$ and $F_2=\left\{\left(u,m_{F_2}{}^P,n_{F_2}{}^P,m_{F_2}{}^N,m_{F_2}{}^N/u\in X\right)\right\}$ be BPFUSS sets then,

(i) $F_1\cup F_2=\left\{\left(u,\max\left(m_{F_1}{}^P,m_{F_2}{}^P\right),\min\left(n_{F_1}{}^P,n_{F_2}{}^P\right),\min\left(m_{F_1}{}^N,m_{F_2}{}^N\right),\max\left(n_{F_1}{}^N,n_{F_2}{}^N\right)\right)/u\in X\right\}$

(ii) $F_1\cap F_2=\left\{\left(u,\min\left(m_{F_1}{}^P,m_{F_2}{}^P\right),\max\left(n_{F_1}{}^P,n_{F_2}{}^P\right),\max\left(m_{F_1}{}^N,m_{F_2}{}^N\right),\min\left(n_{F_1}{}^N,n_{F_2}{}^N\right)\right)/u\in X\right\}$

(iii) $F_1^C=\left\{\left(u,m_F{}^P,n_F{}^P,m_F{}^N,n_F{}^N/u\in X\right)\right\}$

(iv) $F_1\subset F_2=iff\ m_{F_1}{}^P(u)\le m_{F_2}{}^P(u),n_{F_1}{}^P(u)\ge n_{F_2}{}^P(u),m_{F_1}{}^N(u)\ge m_{F_2}{}^N(u),n_{F_1}{}^N(u)\le n_{F_2}{}^N(u).$

3. PROPERTIES OF TEMPORAL PYTHAGOREAN FUZZY SUBGROUPS

In this section, we have proved the following propositions based on the above definitions.

Proposition 3.1: Let A =< $\delta^-{}_A$, λ > be a TPFSG of X. Then that $\delta^-{}_A(x^{-1},t)=\delta^-{}_A(x,t)$ and $\lambda(x^{-1},t)=\lambda(x,t)$ for all x € X and t € T.

Proof: For any x € X and t € T , We have

$\delta^-{}_A(x,t)=\delta^-{}_A((x^{-1})^{-1},t)\ge\delta^-{}_A(x^{-1},t)\ge\delta^-{}_A(x,t)$ and

$\lambda(x,t)=\lambda((x^{-1})^{-1},t)\le\lambda(x^{-1},t)\le\lambda(x,t)$

Hence $\delta^-{}_A(x^{-1},t)=\delta^-{}_A(x,t)$ and $\lambda(x^{-1},t)=\lambda(x,t)$.

Proposition 3.2 : Let A ==< $\delta^-{}_A$, λ > be a TFFSG of X. For any x, y € X and t € T if =< $\delta^-{}_A(xy^{-1},t)==\delta^-{}_A(e,t)$ and $\lambda(xy^{-1},t)=\lambda(e)$, then = $\delta^-{}_A(x,t)=\delta^-{}_A(y,t)$ and $\lambda(x,t)=\lambda(y,t)$.

Proof: Let x, y € X and t € T be such that $\bar{\delta}_A(xy^{-1},t) = \bar{\delta}_A(e, t)$ and $\lambda(xy^{-1},t) = \lambda(e, t)$. Using Proposition 3.1, we get $\bar{\delta}_A(x,t) = \bar{\delta}_A((xy^{-1})\mathbf{y},t) \geq r\min\{\bar{\delta}_A(e,t), \bar{\delta}_A(y, t)$ and $\lambda(x,t) = \lambda(xy^{-1},t) \leq \max\{\lambda(e,t) = \lambda(y, t)\} = \lambda(y, t)$ for all x, y € X and t € T.

Similarly, $\bar{\delta}_A(y,t) \geq \bar{\delta}_A(x, t)$ and $\lambda(y, t) \leq \lambda(x, t)$. Hence the proof.

Theorem 3.3: A TFFS A =< $\bar{\delta}_A$, λ) in X is a TPFSG of X if and only if it satisfies

(i) $\bar{\delta}_A(xy^{-1},t) \geq r\min\{\bar{\delta}_A(x, t), \bar{\delta}_A(y, t)$

(ii) $\lambda(xy^{-1},t) \leq \max\{\lambda(x, t), \lambda(y, t)\}$ for all x, y € X and t € T.

Proof: Assume that A = <$\bar{\delta}_A$, λ> is a TPFSG of X and let x,y € X and t €T. Then

$$\bar{\delta}_A(xy^{-1},t) \geq r\min\{\bar{\delta}_A(x, t), \bar{\delta}_A(y, t)\}$$
$$= r\min\{\bar{\delta}_A(x,t), \bar{\delta}_A(y, t)\} \text{ and}$$
$$\lambda(xy^{-1},t) \leq \max\{\lambda(x, t), \lambda(y^{-1},t)\}$$
$$= \max\{\lambda(x,t), \lambda(y,t)\} \text{ by proposition 1}$$

Conversely, Suppose that (i) and (ii) are valid. If we take y = x in (i) and (ii), then

$$\bar{\delta}_A(e,t) = \bar{\delta}_A(xx^{-1}, t) \geq r\min\{\bar{\delta}_A(x, t), \bar{\delta}_A(x, t)\} = \bar{\delta}_A(x,t)$$
$$\lambda(e,t) = \lambda(xx^{-1}, t) \leq \max\{\lambda(x, t), \lambda(y, t)\} = \lambda(x,t)$$

It follows from (i) and (ii) that

$$\bar{\delta}_A(y^{-1},t) = \bar{\delta}_A(ey^{-1}, t) \geq r\min\{\bar{\delta}_A(e, t), \bar{\delta}_A(y, t)\} = \bar{\delta}_A(y,t)$$
$$\lambda(y^{-1},t) = \lambda(ey^{-1}, t) \leq \max\{\lambda(e, t), \lambda(y, t)\} = \lambda(y,t)$$

so that

$$\bar{\delta}_A(xy,t) = \bar{\delta}_A(x(y^{-1})^{-1}, t) \geq r\min\{\bar{\delta}_A(x, t), \bar{\delta}_A(y^{-1},t)\}$$
$$\geq r\min\{\bar{\delta}_A(x, t), \bar{\delta}_A(y,t)\}$$
$$\lambda(xy, t) = \lambda(x(y^{-1})^{-1}, t) \leq \max\{\lambda(x, t), \lambda(y^{-1},t)\} \leq \max\{\lambda(x, t), \lambda(y,t)\}$$

Therefore A =< $\bar{\delta}_A$, λ) in X is a TPFSG of X.

Theorem 3.4.: If A = < $\bar{\delta}_A$, λ) in X is a TPFSG of X then the set

S = { x € X, t € T / $\bar{\delta}_A(x, t) = \bar{\delta}_A(e, t) = \lambda(x, t) = \lambda(y, t)$ } is a subgroup of X.

Proof: Let x,y € S and t € T . Then

$\bar{\delta}_A(x, t) = \bar{\delta}_A(e, t) = \bar{\delta}_A(y, t)$ and $\lambda(x, t) = \lambda(e, t) = \lambda(y, t)$. It follows from theorem 1 that

$$\bar{\delta}_A(xy^{-1},t) \geq r\min\{\bar{\delta}_A(x, t), \bar{\delta}_A(y, t)\} = \bar{\delta}_A(e,t)$$
$$\lambda(xy^{-1},t) \leq \max\{\lambda(x,t), \lambda(y, t)\} = \lambda(e,t)$$

so from Proposition 3.2 that $\delta^{-}_{A}(xy^{-1},t) = \delta^{-}_{A}(e, t)$ and $\lambda(xy^{-1},t) = \lambda(e, t)$.

Hence $x y^{-1}$ € S, and so S is a subgroup of X.

Definition 3.5 : Let A = < δ^{-}_{A} , λ > in X is a TPFS in a set X, r € [0,1] and (α, β) € [0,1]. The set U (A/[α,β],r) = { x € X and t € T / $\delta^{-}_{A}(x, t) \geq [\alpha, \beta]$, $\lambda(x, t) \leq r$ }

Is called the temporal bi fuzzy level set of A.

Theorem 3.6: For a A = < δ^{-}_{A} , λ) in X, the following or equivalent.

(i) A = < δ^{-}_{A} , λ) is a TPFSG of X.

The non empty temporal Pythagorean fuzzy level set A If A = < δ^{-}_{A} , λ > is a subgroup of X.

Proof: Assume that A = < δ^{-}_{A} , λ) is a TPFSG of X. Let x, y € U (A: [α, β],r) for all r€[0,1], [α, β] € [0,1] and t € T.

Then If $\delta^{-}_{A}(x, t) \geq [\alpha, \beta]$, $\lambda(x, t) \leq r$,

$\delta^{-}_{A}(y, t) \geq [\alpha, \beta]$, $\lambda(y, t) \leq r$,

It follows from proposition 3.1 that

$\delta^{-}_{A}(xy^{-1},t) \geq r \min \{ \delta^{-}_{A}(x, t), \delta^{-}_{A}(y, t) \} \geq [\alpha, \beta]$ and

$\lambda(xy^{-1},t) \leq r \max \{ \lambda(x, t), \lambda(y, t) \} \leq r$ so that xy^{-1} € U (A : [α, β],r).

Therefore A = <δ^{-}_{A} , λ > is a subgroup of X.

Conversely, let r € [0,1] and [α, β] € D[0,1] be such that U (A;[α, β],r) ≠ ∅ and U (A;[α, β],r) is a subgroup of X.

Suppose that theorem 3.1 (i) is not true and theorem 3.1 (ii) is valid. Then there exists $[\alpha_o, \beta_o]$ € D [0,1] and a,b € X and t € T such that

$\delta^{-}_{A}(ab^{-1},t) \leq [\alpha_o, \beta_o] \leq r \min \{\delta^{-}_{A}(a, t), \delta^{-}_{A}(b ,t)\}$ and

$\lambda(ab^{-1},t) \leq \max \{\lambda(a, t), \lambda(b ,t)\}$

It follows that a,b € $U (A;[\alpha_o,\beta_o],\max\{ \lambda(a, t), \lambda(b ,t)\}$ but $ab^{-1} \notin U (A;[\alpha_o,\beta_o],\max\{ \lambda(a,t), \lambda(b ,t)\}$

This is a contradiction. If theorem 3.1 (i) is true and theorem 3.1 (ii) is not valid then

$\delta^{-}_{A}(ab^{-1},t) \geq r \min \{\delta^{-}_{A}(a, t), \delta^{-}_{A}(b ,t)\}$ and

$\lambda(ab^{-1},t) \geq r_o \geq \max \{\lambda(a, t), \lambda(b ,t)\}$ for some r_o € [0,1] and a, b €X, t € T.

thus a, b € $U (A;[\alpha_o, \beta_o],r_o)$ but a, b ∉ U ⊂ A;

r min <{ $\delta^{-}_{A}(a ,t)$, $\delta^{-}_{A}(b ,t)$}, r_o) which is a contradiction.

Assume that there exist $[\alpha_o, \beta_o]$ € D[0,1], r_o € [0,1] and a, b € X such that

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$\delta^{-}_{A}(ab^{-1},t) < [\alpha_o, \beta_o] \leq r\min\{\delta^{-}_{A}(a,t), \delta^{-}_{A}(b,t)\}$ and

$\lambda(ab^{-1},t) > r_o \geq \max\{\lambda(a,t), \lambda(b,t)\}$. Then a,b € U (A;[$\alpha_o, \beta_o$],$r_o$) but

$ab^{-1} \notin$ U (A;[α_o, β_o], r_o). This is also a contradiction. Hence (i) and (ii) of theorem 3.1 are valid. Therefore A = < δ^{-}_{A} , λ> is a TPFSG of X.

Definition 3.7 : Let X and Y be give crisp sets. A mapping of X x T → Y x T induces two mapping C : C(X, t) → C(Y, t) , A = C(A) and C^{-1} : C(Y, t) → C(X, t). B=C^{-1} (B), where

$$C(\delta^{-}_{A}(y,t)) = \begin{cases} r(\text{Sup}\ \delta^{-}_{A}(x,t))_{y=f(x)} & \text{if } f^{-1}(y) \neq \emptyset \\ [0,0] & \text{otherwise} \end{cases}$$

$$C(\lambda(y,t)) = \begin{cases} \inf\ \lambda(x,t))_{y=f(x)} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

For all (y, t) € Y x T and $C^{-1}(\delta^{-}_{B}(x,t)) = \delta^{-}_{B}(f(x),t)$ for all x € X and t € T.

A TBFS A = <δ^{-}_{A} , λ> in X has the property if for any subset P of X there exists xo € P such that

$\delta^{-}_{A}(x_o, t) = r(\sup)_{x \in P}\ \delta^{-}_{A}(x,t)$ and

$\lambda(x_0,t) = (\inf)_{x \in P}\ \lambda(x,t)$

Theorem 3.8: For a homomorphism of f : X x T → Y x T of groups, let C:C(x, t)→C(y, t) and C^{-1} :C(y, t) →C(x,t) be the temporal Pythagorean fuzzy transformation and inverse temporal Pythagorean fuzzy transformation, respectively, induced by f.

(i) If A = < δ^{-}_{A} , λ > € C(x, t) is a TBFSG of X which has the temporal Pythagorean fuzzy property, then C(A) is a TBPSG of Y.

(ii) If B = < δ^{-}_{B} , k > € C(y, t) is a TBPSG of Y, then C^{-1} (B) is a TBPSG of X.

Proof:

(i) Given f(x,t) , f(y, t) € f(x,t), let $x_o \in f^{-1}(f(x,t))$ and $y_o \in f^{-1}(f(y,t))$ be such that

$\delta^{-}_{A}(x_o,t) = r(\text{Sup})_{a \in f^{-1}(f(x_o,t))}\ \delta^{-}_{A}(a,t)$, $\lambda(x_o,t) = (\inf)_{a \in f^{-1}(f(x,t))}\ \lambda(a,t)$

$\delta^{-}_{A}(y_o,t) = r(\text{Sup})_{b \in f^{-1}(f(y_o,t))}\ \delta^{-}_{A}(b,t)$, $\lambda(y_o,t) = (\inf)_{a \in f^{-1}(f(y,t))}\ \lambda(b,t)$

$C\{(\delta^{-}_{A})(f(x,t)f(y,t)\} = r\ \text{Sup}_{z \in f^{-1}(f(x.t)f(y,t))}\ \delta^{-}_{A}(z,t)$

$\geq \delta^{-}{}_{A}(x_o y_o, t)$

$= r \min \{ \delta^{-}{}_{A}(x_o, t), \delta^{-}{}_{A}(z_o, t)$

$= r \min \{ r\,(\mathrm{Sup})_{a \in f^{-1}(f(x,t))}, r\,(\mathrm{Sup})_{b \in f^{-1}(f(y,t))}\ \delta^{-}{}_{A}(b, t)\}$

$= r \min \{C(\delta_A)f(x, t), C(\delta_A)(f(y, t)\}$

$C\{(\delta_A)(f(x, t)^{-1}\}$ $= r\,\mathrm{Sup}_{z \in f^{-1}(f(x.t)^{-1})}\ \delta^{-}{}_{A}(z, t)$

$\geq \delta^{-}{}_{A}(x_o^{-1}, t)$

$\geq \delta^{-}{}_{A}(x_o, t)$

$= C\,(C\,\delta_A)(f(x, t)$

$C\{(C\lambda)f(x, t)^{-1})\}$ $= \inf_{z \in f^{-1}(f(x.t))^{-1}}\ \lambda(z, t)$

$\geq \lambda(x_o^{-1}, t)$

$= \lambda(x_o, t)$

$= C(C\lambda)(f(x, t)$ Therefore C(A) is a TPFSG of Y.

(ii) For any x, y € X and t € T, We have

$C^{-1}\{(C\,\delta_B)((xy, t)\}$ $= (\delta^{-}{}_{B})(f(xy, t)\}$

$= (\delta^{-}{}_{B})(f(x, t)f(y, t)\}$

$= r \min (C^{-1}\{(\delta^{-}{}_{B})(x, t)$

$= C^{-1}\{(\delta^{-}{}_{B})(y, t)\}$

$C^{-1}\{(\delta_B)((x^{-1}, t)\}$ $= (\delta^{-}{}_{B})(f(x^{-1}, t)\}$

$= (\delta^{-}{}_{B})(f(x, t))^{-1}$

$= (\delta^{-}{}_{B})f(x, t)$

$C^{-1}\{(k)((xy,t)\}$ $= k(f(xy, t)$

$= k\,(f(x, t), f(y, t)\}$

$\leq \max\{K\,f(x,t), k\,f(y, t)\}$

$= \max\{C^{-1}(k)(x, t), C^{-1}(k)(y, t)\}$

And

$C^{-1}\{(k)((x^{-1}, t)\}$ $= k(f(x^{-1}, t)$

$= k\,(f(x, t))^{-1}$

$\leq k\,(f(x, t))$

$= C -1\,(k)\,(x, t)$

Thus C^{-1} (B) is a TPFSG of X.

4. ANALYSED TEMPORAL PYTHAGOREAN FUZZY SOFT R-IDEALS

Definition 4.1: A bipolar temporal Pythagorean uncertainty parameterized collections F in X called bipolar temporal Pythagorean uncertainty soft sub algebra of X if it fulfills,

(i) $m_F{}^P(u*v) \geq T\{m_F{}^P(u), m_F{}^P(v)\}$

(ii) $n_F{}^P(u*v) \leq S\{n_F{}^P(u), n_F{}^P(v)\}$

(iii) $m_F{}^N(u*v) \leq S\{m_F{}^N(u), m_F{}^N(v)\}$

(iv) $n_F{}^N(u*v) \geq T\{n_F{}^N(u), n_F{}^N(v)\},\ for all\ u, v \in X.$

Definition 4.2: A Bipolar temporal Pythagorean uncertainty parameterized collections 'F' of a BCK-algebra X is known to be a bipolar temporal Pythagorean uncertainty soft ideal (BPTPUSI) of X, if the subsequent results are satisfied.

(i) $m_F{}^P(0) \geq m_F{}^P(u)\ and\ n_F{}^P(0) \leq n_F{}^P(u)$

(ii) $m_F{}^N(0) \leq m_F{}^N(u)\ and\ n_F{}^N(0) \geq n_F{}^N(u)$

(iii) $m_F{}^P(u) \geq T\{m_F{}^P(u*), m_F{}^P(v)\}\ and\ n_F{}^P(u) \leq S\{n_F{}^P(u*v), n_F{}^P(v)\}$

(iv) $m_F{}^N(u) \leq S\{m_F{}^N(u*v), m_F{}^N(v)\}\ and\ n_F{}^N(u) \geq T\{n_F{}^N(u*v), n_F{}^N(v)\},\ if\ u, v \in X.$

Definition 4.3: A bipolar uncertainty soft set F in X is known as a bipolar temporal Pythagorean uncertainty soft R-ideal (BPTPUSRI) of X if it fulfills,

(i) $m_F{}^P(0) \geq m_F{}^P(u)\ and\ n_F{}^P(0) \leq n_F{}^P(u)$

(ii) $m_F{}^N(0) \leq m_F{}^N(u)\ and\ n_F{}^N(0) \geq n_F{}^N(u)$

(iii) $m_F{}^P(v*u) \geq T\{m_F{}^P(u*w)*(0*v), m_F{}^P(w)\}\ \ and$

$n_F{}^P(v*u) \leq S\{n_F{}^P(u*w)*(0*v), n_F{}^P(w)\}$

(iv) $m_F{}^N(v*u) \leq S\{m_F{}^N(u*w)*(0*v), m_F{}^N(w)\}\ \ and$

$n_F{}^N(v*u) \geq T\{n_F{}^N(u*w)*(0*v), n_F{}^N(w)\},\ for\,all\,u, v, w \in X.$

Example 4.4: We have a BCK-algebra $X = \{\ell, m, n, p\}$ with the following Cayley table.

$*$	l	m	n	p
l	l	m	n	p
m	m	l	p	n
n	n	p	l	m
p	p	n	m	l

Define a BPTPUSS ‘F’ in X by

x	l	m	n	p
$(m_F{}^P, n_F{}^P)$	$[0.2, 0.5]$	$[0.4, 0.6]$	$[0.5, 0.7]$	$[0.2, 0.9]$
$(m_F{}^N, n_F{}^N)$	$[-0.7, -0.1]$	$[-0.9, -0]$	$[-0.4, -0]$	$[-0.7, -0]$

Then, ‘F’ is BPTPUSRI of X.

The consecutive results are the standard results with relevant results.

Hence, ‘F’ is a BPTPUSI of X.

Theorem 4.5: Let F be a BPTPURI of X. Then the collection $\Delta = \{u \in X / m_F{}^N(u) = m_F{}^N(0), n_F{}^N(u) = n_F{}^N(0), m_F{}^P(u) = m_F{}^P(0), n_F{}^P(u) = n_F{}^P(0)\}$ is an R-ideal of X.

Proof: Clearly, $0 \in \Delta$. Let u, v, w ϵ X be such that $((u*w)*(0*v)) \in \Delta$ *and* $w \in \Delta$. Then,

$$m_F{}^N(u) \leq m_F{}^N(v*u) \leq S\{m_F{}^N(u*(0*v)), m_F{}^N(w)\} = m_F{}^N(0)$$

$$n_F{}^N(u) \geq n_F{}^N(v*u) \geq T\{n_F{}^N(u*(0*v)), n_F{}^N(w)\} = n_F{}^N(0)$$

$$m_F{}^P(u) \geq m_F{}^P(v*u) \geq T\{m_F{}^P(u*(0*v)), m_F{}^P(w)\} = m_F{}^P(0)$$

$$n_F{}^P(u) \leq n_F{}^P(v*u) \leq S\{n_F{}^P(u*(0*v)), n_F{}^P(w)\} = n_F{}^P(0)$$

By using definition 2.1 then,

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$m_F{}^N(v*u)=m_F{}^N(0),\ n_F{}^N(v*u)=n_F{}^N(0).$

That is $v*u\in\Delta$. Therefore Δ is R-ideal of X.

Theorem 4.6: If F_1 *and* F_2 are a BPTPUSRI of X, then $F_1\cap F_2$ is also BPTPUSRI of X.

Proof: Now, $m_{F_1}{}^N(0)\le m_{F_1}{}^N(u),\ n_{F_1}{}^N(0)\ge n_{F_1}{}^N(u)$ *and*

$$m_{F_2}{}^N(0)\le m_{F_2}{}^N(u),\ n_{F_2}{}^N(0)\ge n_{F_2}{}^N(u),\ for\,all\,u\in X.$$

$$S\left\{m_{F_1}{}^N(0),\ m_{F_2}{}^N(0)\right\}\le S\left\{m_{F_1}{}^N(u),\ m_{F_2}{}^N(u)\right\}=m_{F_1\cap F_2}{}^N(0)\le m_{F_1\cap F_2}{}^N(0)\ and$$

$$T\left\{m_{F_1}{}^N(0),\ m_{F_2}{}^N(0)\right\}\ge T\left\{m_{F_1}{}^N(u),\ m_{F_2}{}^N(u)\right\}=m_{F_1\cap F_2}{}^N(0)\ge m_{F_1\cap F_2}{}^N(0),\ for\,all\,u\in X.$$

Also,

$$m_{F_1}{}^N(v*u)\le S\left\{m_{F_1}{}^N((u*w)*(0*v)),\ m_{F_1}{}^N(w)\right\}$$

$$m_{F_2}{}^N(v*u)\le S\left\{m_{F_2}{}^N((u*w)*(0*v)),m_{F_2}{}^N(w)\right\}$$

$$n_{F_1}{}^N(v*u)\ge T\left\{n_{F_1}{}^N((u*w)*(0*v)),n_{F_1}{}^N(w)\right\}$$

$$n_{F_2}{}^N(v*u)\ge T\left\{n_{F_2}{}^N((u*w)*(0*v)),n_{F_2}{}^N(w)\right\}$$

$$S\left\{m_{F_1}{}^N(v*u),m_{F_2}{}^N(v*u)\right\}\le S\left\{m_{F_1}{}^N((u*w)*(0*v)),\ m_{F_1}{}^N(w)\right\},\ S\left\{m_{F_2}{}^N((u*w)*(0*v)),\ m_{F_2}{}^N(w)\right\}$$

$$T\left\{n_{F_1}{}^N(v*u),\ n_{F_2}{}^N(v*u)\right\}\ge T\left\{n_{F_1}{}^N((u*w)*(0*v)),\ n_{F_1}{}^N(w)\right\},\ T\left\{n_{F_2}{}^N((u*w)*(0*v)),\ n_{F_2}{}^N(w)\right\}$$

$$m_{F_1}{}^P(0)\ge m_{F_1}{}^P(u),\quad n_{F_1}{}^P(0)\le n_{F_1}{}^P(u)\ and$$

$$m_{F_2}{}^P(0)\ge m_{F_2}{}^P(u),\quad n_{F_2}{}^P(0)\le n_{F_2}{}^P(u),\ for\,all\,u\in X.$$

$$T\left\{m_{F_1}{}^P(0),\ m_{F_2}{}^P(0)\right\}\ge T\left\{m_{F_1}{}^P(u),\ m_{F_2}{}^P(u)\right\}\ =m_{F_1\cap F_2}{}^P(0)\ge m_{F_1\cap F_2}{}^P(u)\ and$$

$$S\left\{n_{F_1}{}^P(0),\ n_{F_2}{}^P(0)\right\}\le S\left\{n_{F_1}{}^P(u),\ n_{F_2}{}^P(u)\right\}\ =n_{F_1\cap F_2}{}^P(0)\le n_{F_1\cap F_2}{}^P(u),\ for\,all\,u\in X.$$

Again,

$$m_{F_1}{}^P(v*u)\ge T\left\{m_{F_1}{}^P((u*w)*(0*v)),\ m_{F_1}{}^P(w)\right\}$$

$$m_{F_2}{}^P(v*u)\ge T\left\{m_{F_2}{}^P((u*w)*(0*v)),\ m_{F_2}{}^P(w)\right\}$$

$$n_{F_1}{}^P(v*u)\le S\left\{n_{F_1}{}^P((u*w)*(0*v)),\ n_{F_1}{}^P(w)\right\}$$

$$n_{F_2}{}^P(v*u)\le S\left\{n_{F_2}{}^P((u*w)*(0*v)),\ n_{F_2}{}^P(w)\right\}$$

$$T\left\{m_{F_1}{}^{N}(v*u),\ m_{F_2}{}^{N}(v*u)\right\} \geq T\left\{T\left\{m_{F_1}{}^{P}((u*w)*(0*v)),\ m_{F_1}{}^{P}(w)\right\},\ T\left\{m_{F_2}{}^{P}((u*w)*(0*v)),\ m_{F_2}{}^{P}(w)\right\}\right\}$$

$$S\left\{n_{F_1}{}^{N}(v*u),\ n_{F_2}{}^{N}(v*u)\right\} \leq S\left\{S\left\{n_{F_1}{}^{P}((u*w)*(0*v)),\ n_{F_1}{}^{P}(w)\right\},\ S\left\{n_{F_2}{}^{P}((u*w)*(0*v)),\ n_{F_2}{}^{P}(w)\right\}\right\}$$

$$m_{F_1\cap F_2}{}^{P}(0) \geq T\left\{m_{F_1\cap F_2}{}^{P}((u*w)*(0*v)),\ m_{F_1\cap F_2}{}^{P}(w)\right\}\ and$$

$$n_{F_1\cap F_2}{}^{P}(0) \leq S\left\{n_{F_1\cap F_2}{}^{P}((u*w)*(0*v)),\ n_{F_1\cap F_2}{}^{P}(w)\right\},\ for\ all\ u,v,w \in X.$$

Hence, F1∩F2 is also BPFUSRI of X.

Definition 4.7: For a bipolar temporal Pythagorean uncertainty soft set 'F' in X and $(\alpha,\ \beta) \in [0,\ 1]$ *and* $(\gamma,\ \sigma) \in [-1,\ 0]$, the positive $(\alpha, \beta)-cut$ and negative $(\gamma,\ \sigma)-cut$ are denoted by $F^{P}(\alpha,\ \beta)$ *and* $F^{N}(\gamma,\ \sigma)$ are expressed as follows:

$$F^{P}(\alpha,\beta)=\left\{a \in X / m_{F}{}^{P}(u) \geq \alpha\ and\ n_{F}{}^{P}(u) \leq \beta\right\} and$$

$$F^{N}(\gamma,\sigma)=\left\{u \in X / m_{F}{}^{N}(u) \geq \gamma\ and\ n_{F}{}^{N}(u) \leq \sigma\right\} with\ \alpha+\beta \leq 1\ and\ \gamma+\sigma \geq -1\ respectively.$$

The bipolar temporal Pythagorean uncertainty soft level cut of F denoted by F_{cut} is represented to be the collections $F_{cut} = \left(F^{P}(\alpha,\beta),\ F^{N}(\gamma,\sigma)\right)$.

Theorem 4.8: A bipolar temporal Pythagorean uncertainty soft set F in X is a BPFUSRI of X iff for all $(\alpha,\beta) \in [0,\ 1]$ and $(\gamma,\ \sigma) \in [-1, 0]$, the non-empty positive $(\alpha,\beta)-cut$ and the non-empty negative $(\gamma,\sigma)-cut$ are BPTFUSRI of X.

Proof: Let 'A' be BPTPUSRI of X and clear that $F^{P}(\alpha,\beta)$ *and* $F^{N}(\gamma,\sigma)$ are non-empty for $(\alpha,\beta) \in [0,1]$ *and* $(\gamma,\sigma) \in [-1,0]$, obviously $0 \in F^{P}(\alpha,\beta) \cap F^{N}(\gamma,\sigma)$.

Let for all u, v, w ϵ X be such that

$$m_{F}{}^{N}((u*w)*(0*v)) \in F^{N}(\gamma,\sigma)\ and\ m_{F}{}^{N}(w) \in F^{N}(\gamma,\sigma)$$

$$n_{F}{}^{N}((u*w)*(0*v)) \in F^{N}(\gamma,\sigma)\ and\ n_{F}{}^{N}(w) \in F^{N}(\gamma,\sigma)$$

Then

$$m_{F}{}^{N}((u*w)*(0*v)) \leq \gamma,\ m_{F}{}^{N}(w) \leq \gamma$$

$$n_{F}{}^{N}((u*w)*(0*v)) \geq \sigma,\ n_{F}{}^{N}(w) \leq \sigma.$$

It follows from definition 2.1 that

$$m_{F}{}^{N}(v*u) \leq S\left\{m_{F}{}^{N}((u*w)*(0*v)), m_{F}{}^{N}(w)\right\} \leq \gamma\ and$$

$$n_F{}^N(v*u) \geq T\left\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\right\} \geq \sigma.$$

So that, $v*u \in F^N(\gamma,\sigma)$.

Now let us see that,

$$m_F{}^P((u*w)*(0*v)) \in F^P(\alpha,\beta) \text{ and } m_F{}^P(w) \in F^P(\alpha,\beta) \text{ and}$$

$$n_F{}^P((u*w)*(0*v)) \in F^P(\alpha,\beta) \text{ and } n_F{}^P(w) \in F^P(\alpha,\beta)$$

Then

$$m_F{}^P((u*w)*(0*v)) \geq \alpha, m_F{}^P(w) \geq \alpha$$

$$n_F{}^P((u*w)*(0*v)) \leq \beta, n_F{}^P(w) \leq \beta.$$

If obeys from the definition 2.1 that

$$m_F{}^P(v*u) \geq T\left\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\right\} \geq \alpha \text{ and}$$

$$n_F{}^P(v*u) \leq S\left\{n_F{}^P((u*w)*(0*v)), n_F{}^P(w)\right\} \leq \beta$$

So that, $v*u \in F^P(\alpha,\beta)$.

Therefore, $F^P(\alpha,\beta)$ *and* $F^N(\gamma,\sigma)$ are R-ideal of X. Reversely, suppose that the non-empty, negative $(\gamma,\sigma)-cut$ and the elements of positive $(\alpha,\beta)-cut$ are R-ideal of X for every $(\alpha,\beta) \in [0,1]$ *and* $(\gamma,\sigma) \in [-1,0]$.

If $m_F{}^N(0) \geq m_F{}^N(u)$, $n_F{}^N(0) \leq n_F{}^N(u)$

$m_F{}^P(0) \leq m_F{}^P(u)$, $n_F{}^P(0) \geq n_F{}^P(u)$, *for* $u \in X$.

Then either $\mathbf{0} \notin F^N\left(m_F{}^N(u), n_F{}^N(u)\right)$ or $\mathbf{0} \notin F^P\left(m_F{}^P(u), n_F{}^P(u)\right)$.

This is a contradiction that $m_F{}^N(0) \leq m_F{}^N(u)$, $n_F{}^N(0) \geq n_F{}^N(u)$ and $m_F{}^P(0) \geq m_F{}^P(u)$, $n_F{}^P(0) \geq n_F{}^P(u)$, *for all* $u \in X$.

Let us assume that,

$$m_F{}^N(v*u) \geq S\left\{m_F{}^N((u*w)*(0*v)), m_F{}^N(w)\right\} = \gamma \text{ and}$$

$$n_F{}^N(v*u) \leq T\left\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\right\} = \sigma \text{ for all } u,v,w \in X.$$

Then, $((u*w)*(0*v)) \in F^N(\gamma,\sigma)$ *and* $w \in F^N(\gamma,\sigma)$, *but* $v*u \notin F^N(\gamma,\sigma)$.

This is not possible and thus,

$m_F{}^N(v*u) \leq S\{m_F{}^N((u*w)*(0*v)), m_F{}^N(w)\} = \gamma$ *and*

$n_F{}^N(v*u) \geq T\{n_F{}^N((u*w)*(0*v)), n_F{}^N(w)\} = \sigma$ *for all* $u, v, w \in X$.

If $m_F{}^P(v*u) \leq T\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\} = \alpha$ *and*

$n_F{}^P(v*u) \geq S\{n_F{}^P((u*w)*(0*v)), n_F{}^N(w)\} = \beta$ *for all* $u, v, w \in X$.

Then, $((u*w)*(0*v)) \in F^P(\alpha, \beta)$ *and* $w \in F^P(\alpha, \beta)$, *but* $v*u \notin F^P(\alpha, \beta)$.

This is not possible and thus,

$m_F{}^P(v*u) \geq T\{m_F{}^P((u*w)*(0*v)), m_F{}^P(w)\} = \alpha$ *and*

$n_F{}^P(v*u) \leq S\{n_F{}^P((u*w)*(0*v)), n_F{}^N(w)\} = \beta$ *for all* $u, v, w \in X$.

Consequently, 'F' in BPTPUSRI of X.

CONCLUSION: we apply the motion of temporal Pythagorean uncertainty soft set of a group, and introduced temporal Pythagorean uncertainty soft set groups. And provide characterization of a TBPSG and study how the images or pre images of TBPSG become TBPSGS.

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