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Applications of (3, 2) - Fuzzy Structures in N-Fuzzy Environment Over Ab-Ideals

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Abstract: In this paper, we propose negative implicative ideals in AB-algebras. Relationship between (3, 2) -uncertainty ideals and (3, 2)- uncertainty structure negative implicative ideals are discussed. Conditions and characterization of (3, 2)- uncertainty negative implicative ideals are provided.

Key words: (3, 2)- fuzzy set, intuitionistic fuzzy set, AB –ideal, negative ideal, AB-algebra, implicative, fuzzy structure, N-ideal.

1. Introduction:

The idea of uncertainty sets was introduced by Zadeh [15]. The concept of uncertainty sets has various applications in real life situations, and many scholars have reached uncertainty set theory. After the introduction of the idea of uncertainty sets, various new finding were studied on the generalizations of uncertainty sets.. The concept of bi fuzzy uncertainty sets indulged by Atanassov [[1],[2],[3]] invented Intuitionistic uncertainty set. Intuitionistic uncertainty subgroup was first studied by Biswas [7]. Yager [12] introduced Pythagorean uncertainty set, where the sum of square of the membership degree and non-membership degree lies between 0 and 1. Pythagorean uncertainty set is more fruitful in many decision making problems. Bhuniaet al [6] represented the notion of Pythagorean uncertainty subgroup (PFSG) as a generalization of Intuitionistic uncertainty subgroup and investigated various properties of Pythagorean uncertainty subgroup. Also, they introduced Pythagorean uncertainty cosset and Pythagorean uncertainty normal subgroup (PUNSG) with their properties. Further, they define the notion of Pythagorean uncertainty level subgroup and establish related properties of it. Senapti and Yager [9] coined fermatean uncertainty set (FFS) with its comparison measures. We have developed some new operators for Fermatean uncertainty sets. Silambarasan [11] Fermatean uncertainty set gives a modern way to model vagueness and uncertainty with high precision and accuracy compared to Intuitionistic uncertainty set. Balamurugan and Nagarajan [[5],[6]] discussed the concept of fermatean fuzzy implicative P-ideal structure in KU-algebra .Application of IUS's appear in several fields, including medical diagnosis, optimization problems and multi criteria decision making. Yager [[13],[14]] studied a new uncertainty collection called a Pythagorean uncertainty set. Fermatean uncertainty collections were introduced by Senapti and Yager [9], and they also explained basic operation over the Fermatean uncertainty collections. The concept of (3,2)- uncertainty set proposed by [8]. This is extended in implicative ideal structures by [8]. In this paper, we propose negative implicative ideals in AB-algebras. Relationship between (3,2)- uncertainty ideals and (3,2)- uncertainty structure negative implicative ideals are discussed. Conditions and characterization of (3, 2)- uncertainty negative implicative ideals are provided.

2. PRELIMINARIES

In this part, we will discuss some concepts related to AB-algebras and (3, 2)- Negative fuzzy sets.

Definition 2.1: Let X be as set with binary operation $*$and a constant 0 .The (X, * , 0) is called an AB-algebra if the following axioms are satisfied for any $x, y, z \in X$,

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(AB_1): $(x*y)*(z*y)*(x*z) = 0$, (AB_2): $x*0 = 0$, (AB_3): $0*x = 0$

In what follows, let $(X,*,0)$ denote AB-algebras unless otherwise specified. For brevity, we also call X a AB-algebras. In X, we can define a binary relation $\leq$ by $x \leq y$ if and only if $y*x = 0$.

Definition 2.2: $(X,*,0)$is AB-algebra if and only if it satisfies

(AB_5): $(y*z)*(x*z) \leq (x*y)$, (AB_6): $0 \leq x$.

Example 2.3: Let X = {0,1,2,3} be a given set and * be the binary operator. Then

Table – 1

·	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

Clearly, By routine calculations, it is easy to see that X is AB-algebra.

Definition 2.4: In a AB-algebra, the following identities are true:

(i) $z*z = 0$

(ii) $z*(x*z) = 0$

(iii) $z*(y*x) = y*(z*x), for\ all\ x,y,z \in X$

By a BCK-algebra, we mean a AB-algebra X satisfying the condition

$$(\forall x \in X)\ (\ (x*x) = 0\ ).$$

A non-empty subset A of a AB-algebra X is called AB-ideal of X if it satisfies the following conditions.

$$(i)\ 0 \in A$$
$$(ii)(\forall x,y \in X)\ (x*y \in A,\ \ y \in A\ \Rightarrow x \in A) \quad ------(1)$$

Let A be a subset of BCK-algebra. Then 'A' is called implicative AB-ideal of X if the condition (i) of equation-1 holds and the following ascertain is valid.

$$(\forall x,y,z \in X)\ (x*y^{n+1}*z \in A\ \ and\ \ z \in A\ \Rightarrow x*y^n \in A)$$

Any implicative AB-ideal is a AB-ideal, but the converser is not true.

Example 2.5: Let $X = \{0,1,2,3\}$ is which $*$ is defined by table – 1. Clearly $(X,*,0)$ is a AB-algebra. It is easy to show that $A = \{0,1\}\ and\ B = \{0,1,2,3\}$ are AB-ideals of X.

Lemma 2.6: A subset A of a AB-algebra X is an implicative AB-ideal of X if and only if A is a AB-ideal of X which satisfies the following condition.

$$(\forall x,y \in X)\ ((x*y^{n+1})*y \in A\ \ \Rightarrow x*y^{n+1} \in A)$$

We denote the collection of functions from a set X to $[0,-1] by\ F(X,[-1,0])$. An element of $F(X,[-1,0])$ is called a negative-valued function X to $[-1,0]$ $(briefly\ N-function\ on\ X)$. An ordered pair $(X,l)\ of\ X$ and an N-function $'l'\ on\ X$ is called N-structure.

Definition 2.7: An Intuitionistic fuzzy set on the universe X is an object of the form $A = \{(x,\ \alpha_A(x), \beta_A(x)\ /x \in X\ \}$,where$\alpha_A(x) \in [-1,0]$ is called"degree of positive membership of x in A" and $\beta_A(x) \in [-1,0]$is called "degree of negative membership of x in A", and Where α_A, $\beta_A$satisfies the condition;$-1 \leq \alpha_A(x) + \beta_A(x) \leq 0, \forall x \in X$.

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The family of all Intuitionistic negative fuzzy set in X is denoted by INFS(X).The complement of a INFS(X).of A is denoted by$A^C = \{x,\ \beta_A(x), \alpha_A(x)/x \in X\}$. Formally, a (INFS(X).associate two fuzzy sets $x, P_A(x),\ \beta_A(x)$
$\alpha_A : X \to [-1,0]; \beta_A : X \to [-1,0]$ and$-1 \le \alpha_A(x) + \beta_A(x) \le 0,\ \forall x \in X.$
Obviously, any INFS(X).$A = \{(x,\ \alpha_A(x), I_A(x))\}$ may be identified with the INFS in the form $A = \{(x,\ \alpha_A(x), \beta_A(x), 0)\ /x \in X\}$.
The operations on INFS(X) are introduced,

$(i)\quad \forall A, B \in INFS(X).$

$(ii)\ A \subseteq B\ if\ and\ only\ if\ \alpha_A(x) \le \alpha_B(x), \beta_A(x) \ge \beta_B(x) \forall x \in X,$

$(iii)\ A = B\ if\ and\ only\ if\ A \subseteq B\ and\ B \subseteq A.$

$(iv)\ A \cup B = \{(x, max(\alpha_A(x), \alpha_B(x)), min(\beta_A(x), \beta_B(x))/x \in X\}$

$(v)\ A \cap B = \{(x, min(\alpha_A(x), \alpha_B(x)), max(\beta_A(x), \beta_B(x))/x \in X\}.$

The concept of (3, 2)- uncertainty set proposed by [15].
To illustrate the importance of (3, 2)- uncertainty collection to extend the grade of membership and non-membership degrees, assume that $\alpha_D(x) = 0.9\ and$
$\beta_D(x) = 0.8$ for $X = \{x\}$. We obtain$0.9 + 0.8 = 1.7 > 1$,
$(0.9)^2 + (0.8)^2 = 1.45 > 1$ And $(0.9)^3 + (0.8)^3 = 1.241 > 1$ which means
That $D = (0.9, 0.8)$ neither following the condition of Fermatean uncertainty set nor follows the condition of Pythagorean uncertainty set.

On the other hand, $(0.9)^3 + (0.8)^2 = 0.793 < 1$ which mean we can apply the (3, 2)- Uncertainty set to control it. That is, $D = (0.9, 0.8)$ is a (3, 2)- uncertainty set.

Definition 2.8: Let X be a universal set. Then the Fermat's uncertainty set (briefly, Fermat's fuzzy set D is defined by the following;

$D = \{\langle x,\ \alpha_D(x), \beta_D(x)\rangle / x \in X\}$------------(1)

where $\alpha_D : X \to [0,1]$ is the degree of membership and $\beta_D : X \to [0,1]$ is the degree of non – membership of $x \in X$ to D, with the condition
$0 \le (\alpha_D(x))^3 + (\beta_D(x))^2 \le 1$------------(2) the degree of indeterminacy of $x \in X$ to D is defined by $\pi_D(x) = \sqrt[5]{1 - [(\alpha_D(x))^3 + (\beta_D(x))^2]}$.------------(3) .$I$t is clear that $(\alpha_D(x))^3 + (\beta_D(x))^2 + (\pi_D(x))^5 = 1$ and $\pi_D(x) = 0$ whenever
$(\alpha_D(x))^3 + (\beta_D(x))^2 = 1$. In the case of simplicity, we shall mention the symbol$D = (\alpha_D, \beta_D)$ for the (3,2)-uncertainty set $D = \{(x, (\alpha_D(x), \beta_D(x))/x \in X\}$. Here, $\alpha_D{}^3(x) = (\alpha_D(x))^3$ and $\beta_D{}^2(x) = (\beta_D(x))^2$for all $x \in X$.

Example- 2.9: Let D be (3,2)-FS and $x \in X$ such that $\beta_D(x) = 0.82$ and $\pi_D(x) = 0$.Then,

$$|\alpha_D(x)| = \sqrt[3]{|(\beta_D(x) - 1)(\beta_D(x) + 1)|}$$
$$= \sqrt[3]{|(-0.18)(1.82)|}$$
$$= \sqrt[3]{0.3276}$$

In 2013, Yager defined Pythagorean uncertainty subset (PUS) as a generalization of intuitionistic uncertainty set (IUS).

Definition 2.10: An (3,2)- negative fuzzy set on the universe X is an object of the form $A = \{(x,\ \alpha_A(x), \beta_A(x)\ /x \in X\ \}$,where$\alpha_A(x) \in [-1,0]$is called"degree of positive membership of x in A" and $\beta_A(x) \in [-1,0]$is called "degree of negative membership of x in A", and Where α_A , $\beta_A$satisfies the condition;$-1 \leq \alpha_A(x) + \beta_A(x) \leq 0, \forall x \in X$.

3. (3, 2)- NEGATIVE FUZZY SET IMPLICATIVE IDEALS

In what follows, let 'X' denoted a AB-algebra unless otherwise specified.

Definition 3.1: Let X_N be (3,2)- Negative fuzzy set in X.Then X_N is called an (3,2)- Negative fuzzy ideal of X if the following conditions holds.

$$(i)\alpha_D{}^3(0) \leq \alpha_D{}^3(x) \leq max\{\alpha_D{}^3(x * y^{n+1}),\ \alpha_D{}^3(y)\}$$
$$(ii)\beta_D{}^2(0) \geq \beta_D{}^2(x) \geq min\{\beta_D{}^2(x * y^{n+1}),\ \beta_D{}^2(y)\} \qquad ---------(2)$$
$$for\ all\ x, y\ \in X.$$

Definition 3.2: A (3, 2)- Negative fuzzy set X_N over X is called an (3,2)- Negative fuzzy implicative ideal of X if the following assertions are valid.

$$(i)\alpha_D{}^3(0)\ \leq \alpha_D{}^3(x) \leq max\alpha_D\{\alpha_D{}^3((x * y^{n+1}) * z),\ \alpha_D{}^3(y * z)\}$$
$$(ii)\beta_D{}^2(0) \geq \beta_D{}^2(x) \geq min\{\beta_D{}^2((x * y^{n+1}) * z),\ \beta_D{}^2(y * z)\} \qquad ---------(3)$$
$$for\ all\ x, y, z\ \in X.$$

Example 3.3: Let $X = \{0,1,2,3,4\}$ be aAB-algebra. Then Cayley table for the binary operation $*$ is given by the following table.

·	**0**	**1**	**2**	**3**
0	0	0	0	0
1	1	0	0	1
2	2	2	0	2
3	3	3	3	0

Let$X_U = \{(0,\ (-0.8,\ -0.3\ )), (1,\ (-0.6,\ -0.5)), (2,\ (-0.5,\ -0.8)),\ (3,\ (-0.1,\ -0.5)), (4, (-0.2,\ -0.7))\}$ be a (3,2)- negative fuzzy set over X. Then X_N is a (3, 2)- negative fuzzy set implicative N-ideal of X. If we take $z = 0$ in definition 3.2 and using equation (1) then we have the following theorem.

Theorem 3.4: Every (3, 2)- negative fuzzy set implicative ideal is a (3,2)- negative fuzzy set ideal. The following example shows that the converse of theorem 3.4 does not holds. Let $X = \{0, p, q, r\}$ be a AB-algebra with the Cayley table.

*	0	p	q	r
0	0	0	0	0
p	p	0	0	p
q	q	p	0	q
r	r	r	r	0

Let $X_N = \{(0,\ (c_0, i_2)),\ (p,\ (c_1, i_1)),\ (q,\ (c_1, i_1)),\ (s,\ (c_2, i_0))\}$ be a (m, n)-negative fuzzy structure over X, where $c_0 < c_1 < c_2, i_0 < i_1 < i_2, l_0 < l_1 < l_2\ in\ [-1,0]$. Then X_N is a (3, 2)-negative fuzzy set ideal of X. But it is not (3, 2)-negative fuzzy set implicative ideal of X.

Since,

$$\alpha_D{}^3(q * p) = \alpha_D{}^3(p) = c_1 \nleq c_0 = max\{\alpha_D{}^3((q * p) * p),\ \alpha_D{}^3(p * p)\}$$
$$\beta_D{}^2(q * p) = \beta_D{}^2(p) = i_1 \ngeq i_2 = min\{\beta_D{}^2((q * p) * p),\ \beta_D{}^2(p * p)\}$$

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Definition 3.5: Given an (3, 2)- negative fuzzy structure X_N over X and $r,s \in [-1,0]\ with\ -1 \le r+s \le 0$, we define the following sets.

$\alpha_D{}^{3^r} : \{x \in X/\alpha_D(x) \le r\}$,$\beta_D{}^{3^r} : \{x \in X /\beta_D(x) \ge s\}$

Then we say that the set $X_N(r,s) = \{x \in X / \alpha_D{}^3(x) \le r, \beta_D{}^s(x) \ge s\}$is the (r,s)- level set of X_N .obviously, we have $X_N(r,s) = \alpha_D{}^3r \cap \beta_D{}^s$.

Theorem 3.6: If X_N is a (3, 2)- negative fuzzy implicative N-ideal of X, then $\alpha_D{}^r, \beta_D{}^s$, are $r,s \in [-1,0]\ with\ -1 \le r+s \le 0$ whenever they are non-empty.

Proof: Assume that $\alpha_D{}^r, \beta_D{}^s$are non-empty for all $r,s \in [-1,0]\ with -1 \le r+s \le 0$, then $x \in \alpha_D{}^r,\ y \in \beta_D{}^s\ and\ \ for\ some\ x,y,z \in X$.

Thus, $\alpha_D{}^3(0) \le \alpha_D{}^3(x) \le r, \beta_D{}^2(0) \ge \beta_D{}^2(x) \ge s$.(ie) $0 \in \alpha_D{}^r \cap \beta_D{}^s$.

Let $(x * y^{n+1}) * z \in \alpha_D{}^r\ and\ y * z \in \alpha_D{}^r$.

Then, $\alpha_D{}^3((x * y^{n+1}) * z) \le r\ and\ \alpha_D{}^3(y * z) \le r,$

This implies that $\alpha_D{}^3(x * z) \le max\{\alpha_D{}^3((x * y^{n+1}) * z,\ \alpha_D{}^3(y * z))\} \le r,$

(ie) $x * z \in \alpha_D{}^r$.

If $(u * v^{n+1}) * w \in \beta_D{}^s\ and\ v * w \in \beta_D{}^s,\ then\ \beta_D{}^2((u * v^{n+1}) * w) \ge s\ and$ $\beta_D{}^2(v * w) \ge s$.

Thus, $\beta_D{}^2(u * w) \ge min\{\beta_D{}^2((u * v^{n+1}) * w),\ \beta_D{}^2(v * w) \ge s\},\ and\ so\ u * w \in \beta_D{}^s$.

Therefore, $\alpha_D{}^r, \beta_D{}^s$ are (3,2)- negative fuzzy implicative ideals of X.

Corollary 3.7: Let X_N be a (3,2)- negative fuzzy structure over X and let $r,s \in [-1,0]$ be such that $-1 \le r+s \le 0$. If X_N is a (3,2)- negative fuzzy implicative N-ideal of X, then the non-empty (r,s)- level set of X_N is a (3,2)- negative fuzzy implicative ideal of X.

Proof: Straight forward. The following example illustrates theorem 3.6.

Example 3.8: Let $X = \{0,a,b,c,d\}$ be a AB-algebra with the Cayley table.

*	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Let$X_N = \{(0,\ (-0.9,\ -0.3)), (a,\ (-0.7,\ -0.2)), (b,\ (-0.5,\ -0.5)),\ (c,\ (-0.4,$ $-0.3)), (d, (-0.3,\ -0.2))\}$ be an (3, 2)- negative fuzzy structure over X. By routine calculations, we can show that X_N is an (3, 2)--negative fuzzy implicative ideal of X.

Then, $$\alpha_D{}^r = \begin{cases} \varphi & if\ r \in [-1,\ -0.9] \\ \{0\} & if\ r \in [-0.9,\ -0.7] \\ \{0,a\} & if\ r \in [-0.7,\ -0.5] \\ \{0,a,b\} & if\ r \in [-0.5,\ -0.4] \\ \{0,a,b,c\} & if\ r \in [-0.4, -0.3] \\ X & if\ r \in [-0.3, 0] \end{cases}$$

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$$\beta_D{}^s = \begin{cases} \varphi & if\ s \in [-0.3, -0.2] \\ \{0\} if\ s \in [-0.2, -0.5] \\ \{0, b\} if\ s \in [-0.5, -0.4] \\ \{0, b, c\} & if\ s \in [-0.4, -0.3] \\ \{0, a, b, c\} & if\ s \in [-0.3, -1] \\ X & if\ s \in [-1, 0] \end{cases}$$

And which are (3, 2)- negative fuzzy implicative ideals of X.

Lemma 3.9: Every (3, 2)- negative fuzzy AB ideal of X_N of X satisfies the following assertions; $(\forall\, x, y \in X)$.

We discuss conditions for a (3, 2)- negative fuzzy ideal to be a (3,2)- negative fuzzy AB ideal.

Theorem 3.10: If X_N be a (3, 2)- negative fuzzy ideal of X, then X_N is a (3,2)- negative fuzzy implicative ideal of X if and only if the following assertion is valid.

$$\alpha_D{}^3(x * y^{n+1}) \leq \alpha_D{}^3((x * y^{n+1}) * y),$$
$$\beta_D{}^2(x * y^{n+1}) \geq \beta_D{}^2((x * y^{n+1}) * y), \qquad ----(4)$$
$$for\ all\ x, y \in X.$$

Proof: Assume that X_N is a (3, 2)- negative fuzzy implicative AB-ideal of X. If z is replaced yin definition 3.2, then

$$\alpha_D{}^3(x * y^{n+1}) \leq max\{\alpha_D{}^3\big((x * y^{n+1}) * y\big),\ \alpha_D{}^3(y * y)\}$$
$$= max\{\alpha_D{}^3\big((x * y^{n+1}) * y\big),\ \alpha_D{}^3(0)\} = \alpha_D{}^3\big((x * y^{n+1}) * y\big). \beta_D{}^2(x * y^{n+1})$$
$$\geq min\{\beta_D{}^2\big((x * y^{n+1}) * y\big),\ \beta_D{}^2(y * y)\}$$
$$= min\{\beta_D{}^2\big((x * y^{n+1}) * y\big),\ \beta_D{}^2(0)\} = \beta_D{}^2\big((x * y^{n+1}) * y\big).$$
by using definition 3.2

Conversely, let X_N be a (3, 2)- negative fuzzy ideal of X satisfying theorem 3.10.

Since, $\big((x * z) * z\big) * (y * z) \leq (x * z) * y = (x * y^{n+1}) * z$

We have,
$$\alpha_D{}^2\Big(\big((x * z) * z\big) * (y * z)\Big) \leq \alpha_D{}^2\big((x * y^{n+1}) * z\big)$$
$$\beta_D{}^2\Big(\big((x * z) * z\big) * (y * z)\Big) \geq \beta_D{}^2\big((x * y^{n+1}) * z\big) for\ all\ x, y, z \in X.$$

By Lemma 3.9, It follows from definition 3.1 and definition 3.2 that

$$\alpha_D{}^3(x * z) \leq \alpha_D{}^3\big((x * z) * z\big)$$
$$\leq max\Big\{\alpha_D{}^3\Big(\big((x * z) * z\big) * (y * z)\Big), \alpha_D{}^3(y * z)\Big\}$$
$$\leq max\{\alpha_D{}^3\big((x * y^{n+1}) * z\big), \alpha_D{}^3(y * z)\}, \beta_D{}^2(x * z) \geq \beta_D{}^2\big((x * z) * z\big)$$
$$\geq min\Big\{\alpha_D{}^3\Big(\big((x * z) * z\big) * (y * z)\Big), I_N(y * z)\Big\}$$
$$\geq min\{\beta_D{}^2\big((x * y^{n+1}) * z\big), \beta_D{}^2(y * z)\}$$

Therefore, X_N is a (3, 2)- negative fuzzy implicative N-ideal of X.

Lemma 3.11: For any (3, 2)- negative fuzzy ideal of X_N of X, we have

$$x * y \leq z \ \Rightarrow \begin{cases} \alpha_D{}^3(x) \leq max\{\alpha_D{}^3(y), \alpha_D{}^3(z)\}, \\ \beta_D{}^2(x) \geq min\{\beta_D{}^2(y), \beta_D{}^2(z)\}, \end{cases} ---------(5)$$
$$for\ all\ x, y, z \in X.$$

Lemma 3.12: If an (3, 2)- negative fuzzy structure X_N over X satisfies the Lemma 3.11, then X_N is an (3,2)- negative fuzzy AB ideal of X.

Proof: Since, $0 * x \leq x,\ for\ all\ x \in X,$

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We have, $(0) \le \alpha_D{}^3(x),\ \beta_D{}^3(0) \ge \beta_D{}^s(x) \quad for\ all\ x \in X\ \ by\ Lemma\ 3.11$
Note that $x * (x * y^{n+1}) \le y,\ for\ all\ x, y \in X.$
It follows from Lemma 3.11 that

$$\alpha_D{}^3(x) \le max\{\alpha_D{}^3(x * y^{n+1}), \alpha_D{}^3(y)\}$$

$\beta_D{}^2(x) \ge min\{\beta_D{}^2(x * y^{n+1}), \beta_D{}^2(y)\}$ and
Therefore, X_N is an (3,2)-negative fuzzy AB ideal of X.

Theorem 3.13: For any (3,2)- negative fuzzy structure X_N over X, the following assertions are equivalent.

(i) $X_N$isan (3,2)- negative fuzzy implicative N-ideal of X.
(ii) X_N satisfies the following condition

$$((x * y^{n+1}) * y) * a \le b \implies \begin{cases} \alpha_D{}^3(x * y) \le max\{\alpha_D{}^3(a), \alpha_D{}^3(b)\} \\ \beta_D{}^2(x * y) \ge min\{\beta_D{}^2(a), \beta_D{}^2(b)\} \end{cases} ------(6)$$

$$for\ all\ x, y, a, b \in X.$$

Proof: Suppose that X_N is an (3,2)- negative fuzzy implicative N-ideal of X, then X_N is an (3,2)- negative fuzzy ideal of X by theorem 3.4.
Let $x, y, a, b \in X$ be such that $((x * y^{n+1}) * y) * a \le b,\ then$
$\alpha_D{}^3(x * y) \le \alpha_D{}^3((x * y^{n+1}) * y) \le max\{\alpha_D{}^3(a), \alpha_D{}^3(b)\}$,
$\beta_D{}^2(x * y) \ge \beta_D{}^2((x * y^{n+1}) * y) \ge min\{\beta_D{}^2(a), \beta_D{}^2(b)\}$,
by theorem 3.10 and Lemma 3.12.
Conversely, let X_N is an (3, 2)- negative fuzzy structure over X that satisfies the theorem 3.7.13. Let $x, a, b \in X$ be such that $x * a \le b,\ then((x * 0^{n+1}) * 0) * a \le b,\ and\ so$
$\alpha_D{}^3(x) = \alpha_D{}^3(x * 0) \le max\{\alpha_D{}^3(a), \alpha_D{}^3(b)\}$,
$\beta_D{}^2(x) = \beta_D{}^2(x * 0) \ge min\{\beta_D{}^2(a), \beta_D{}^2(b)\}$,
Hence, X_N is an (3, 2)- negative fuzzy ideal of X, by Lemma 3.12.
Since, $((x * y^{n+1}) * y) * (x * y^{n+1}) * y \le 0$, it follows from theorem 3.13 and definition 3.1 that $\alpha_D{}^3(x * y) \le max\{\alpha_D{}^3((x * y^{n+1}) * y), \alpha_D{}^3(0)\} = \alpha_D{}^3((x * y^{n+1}) * y)$,

$$\beta_D{}^2(x * y) \ge min\{\beta_D{}^2((x * y^{n+1}) * y), \beta_D{}^2(0)\} = \beta_D{}^2((x * y^{n+1}) * y),$$

$for\ all\ x, y \in X$..Therefore, X_N is an (3, 2)- negative fuzzy implicative ideal of X by theorem 3.7.10.

Lemma 3.14: Let X_N be an (3, 2)- negative fuzzy structure over X and assume that $\alpha_D{}^r$, $\beta_D{}^s$and are ideals of X, for all $r, s \in [-1,0]\ with\ -1 \le r + s \le 0$.Then$X_N$ is an (3,2)- negative fuzzy ideal of X.

Theorem 3.15: Let X_N be an (3, 2)- negative fuzzy structure over X and assume that $\alpha_D{}^r$, $\beta_D{}^s$are (3,2)- negative fuzzy AB-ideals of X, for all $r, s \in [-1,0]\ with$ $-1 \le r + s \le 0$ .Then X_N is an (3, 2)- negative fuzzy implicative ideal of X.
Proof: If $\alpha_D{}^r$, $\beta_D{}^s$are negative AB-ideals of X, then $\alpha_D{}^r, \beta_D{}^s$are ideals of X.
Thus, X_N is an (3, 2)- negative fuzzy ideals of X, by Lemma 3.14.
Let $x, y \in X\ and\ r, s \in [-1,0]\ with\ -1 \le r + s \le 0\ such\ that$
$\alpha_D{}^3((x * y^{n+1}) * y) = r$,$\beta_D{}^2((x * y^{n+1}) * y) = s$ and
Then, $(x * y^{n+1}) * y \in \alpha_D{}^r \cap \beta_D{}^s$
Since, $\alpha_D{}^r \cap \beta_D{}^s$ is an (3, 2)- negative fuzzy AB- ideal of X, it follows from Lemma 3.6.7 that $x * y \in \alpha_D{}^r \cap \beta_D{}^s$
Hence, $\alpha_D{}^3(x * y) \le r = \alpha_D{}^3((x * y^{n+1}) * y), \beta_D{}^2(x * y) \ge s = \beta_D{}^2((x * y^{n+1}) * y)$,
Therefore, X_N is a negative fuzzy AB- ideal of X by theorem 3.10.

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Lemma 3.16: Let X_N be an (3, 2)- negative fuzzy AB- ideal of X. Then X_N satisfies the condition of theorem 3.10 if and only if it satisfies the following conditions.

$$\alpha_D{}^3\big((x*z)*(y*z)\big) \le \alpha_D{}^3\big((x*y^{n+1})*z\big),$$
$$\beta_D{}^2\big((x*z)*(y*z)\big) \ge \beta_D{}^2\big((x*y^{n+1})*z\big), \forall\, x,y,z \in X.$$

Corollary 3.17: Let X_N be an (3,2)- negative fuzzy AB-ideal of X, then X_N is an (3,2)- negative fuzzy AB-ideal of X if and only if X_N satisfies Lemma 3.16.
Proof: It follows from theorem 3.10 and Lemma 3.16.

Theorem 3.18: For any (3,2)- negative fuzzy structure X_N over X, the following assertions are equivalent.

(i) X_N is an (3,2)- negative fuzzy AB-ideal of X.

(ii) X_N satisfies the following condition

$$\big((x*y^{n+1})*z\big)*a \le b \implies \begin{cases}\alpha_D{}^3\big((x*z)*(y*z)\big) \le max\{\alpha_D{}^3(a), \alpha_D{}^3(b)\} \\ \beta_D{}^2\big((x*z)*(y*z)\big) \ge min\{\beta_D{}^2(a), \beta_D{}^2(b)\}\end{cases} ----(7)$$
$$for\ all\ x,y,z,a,b \in X.$$

Proof: Suppose that X_N is an (3,2)- negative fuzzy AB-ideal of X, then X_N is an (3,2)- negative fuzzy ideal of X by theorem 3.4.
Let $x,y,z,a,b \in X$ be such that $\big((x*y^{n+1})*z\big)*a \le b$.
Using Corollary 3.14 and Lemma 3.11, we have

$$\alpha_D{}^3\big((x*z)*(y*z)\big) \le \alpha_D{}^3\big((x*y^{n+1})*z\big) \le max\{\alpha_D{}^3(a), \alpha_D{}^3(b)\},$$
$$\beta_D{}^2\big((x*z)*(y*z)\big) \ge \beta_D{}^2\big((x*y^{n+1})*y\big) \ge min\{\beta_D{}^2(a), \beta_D{}^2(b)\},$$
$$for\ all\ x,y,z,a,b \in X.$$

Conversely, let X_N is an (3, 2)- negative fuzzy structure over X that satisfies the theorem 3.18. Let $x,y,a,b \in X$ be such that $\big((x*y^{n+1})*y\big)*a \le b,\ then$

$$\alpha_D{}^3(x*y) = \alpha_D{}^3\big((x*y)*(y*y)\big) \le max\{\alpha_D{}^3(a), \alpha_D{}^3(b)\},$$
$$\beta_D{}^2(x*y) = \beta_D{}^2\big((x*y)*(y*y)\big) \ge min\{\beta_D{}^2(a), \beta_D{}^2(b)\},$$

by theorem 3.4 and theorem 3.18.
Hence, X_N is an (3, 2)- negative fuzzy AB-ideal of X, by theorem 3.13.

Theorem 3.19: LetX_N be (3, 2)- negative fuzzy structure over X, then X_N is an (3,2)- negative fuzzy AB- of X if and only if X_N satisfies definition 2.2 and

$$\alpha_D{}^3(x*y) \le max\left\{\alpha_D{}^3\left(\big((x*y^{n+1})*y\big)*z\right),\ \alpha_D{}^3(z)\right\},$$
$$\beta_D{}^2(x*y) \ge min\left\{\beta_D{}^2\left(\big((x*y^{n+1})*y\big)*z\right),\ \beta_D{}^2(z)\right\},$$
$$for\ all\ x,y,z \in X.$$

Proof: Assume that X_N is an (3, 2)- negative fuzzy AB-ideal of X, then X_N is an (3,2)- negative fuzzy AB-ideal of X by theorem 3.4, and so the condition by definition 3.2 is valid.
Using definition 3.1, equation (1) and Lemma 3.16, we have

$$\alpha_D{}^3(x*y) \le max\{\alpha_D{}^3\big((x*y^{n+1})*z\big),\ \alpha_D{}^3(z)\},$$
$$= max\left\{\alpha_D{}^3\left(\big((x*z)*y^{n+1}\big)*(y*y)\right), \alpha_D{}^3(z)\right\},$$
$$\le max\left\{\alpha_D{}^3\left(\big((x*z)*y^{n+1}\big)*y\right),\ \alpha_D{}^3(z)\right\},$$
$$= max\left\{\alpha_D{}^3\left(\big((x*y)*y^{n+1}\big)*z\right),\ \alpha_D{}^3(z)\right\},$$
$$\beta_D{}^2(x*y) \ge min\{\beta_D{}^2\big((x*y^{n+1})*z\big),\ \beta_D{}^2(z)\},$$

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$$= \min\left\{{\beta_D}^2\left(\left((x * z) * y^{n+1}\right) * (y * y)\right), {\beta_D}^2(z)\right\},$$
$$\geq \min\left\{{\beta_D}^2\left(\left((x * z) * y^{n+1}\right) * y\right),\ {\beta_D}^2(z)\right\},$$
$$= \min\left\{{\beta_D}^2\left(\left((x * y) * y^{n+1}\right) * z\right),\ {\beta_D}^2(z)\right\}, \text{for all } x, y, z \in X$$

Therefore, theorem 3.19 is valid.

Conversely, if X_N is an (3, 2)- negative fuzzy structure over X satisfying two conditions definition 3.6.2 and theorem 3.19, then

$${\alpha_D}^3(x) = {\alpha_D}^3(x * 0) \leq \max\left\{{\alpha_D}^3\left(\left((x * 0) * 0\right) * z\right), {\alpha_D}^3(z)\right\},$$
$$= \max\{{\alpha_D}^3(x * z), {\alpha_D}^3(z)\}$$
$${\beta_D}^2(x) = {\beta_D}^2(x * 0) \geq \min\left\{{\beta_D}^2\left(\left((x * 0) * 0\right) * z\right), {\beta_D}^2(z)\right\},$$
$$= \min\{{\beta_D}^2(x * z), {\beta_D}^2(z)\}$$

Hence, X_N is an (3, 2)- negative fuzzy AB- ideal of X.

Now, if we take $z = 0$ in theorem 3.19 and definition 3.1, then

$${\alpha_D}^3(x * y) \leq \max\left\{{\alpha_D}^3\left(\left((x * y^{n+1}) * y\right) * 0\right), {\alpha_D}^3(0)\right\},$$
$$= \max\{{\alpha_D}^3\left((x * y^{n+1}) * y\right), {\alpha_D}^3(0)\} = {\alpha_D}^3\left((x * y^{n+1}) * y\right)$$
$${\beta_D}^2(x * y) \geq \min\left\{{\beta_D}^2\left(\left((x * y^{n+1}) * y\right) * 0\right), {\beta_D}^2(0)\right\},$$
$$= \min\{{\beta_D}^2\left((x * y^{n+1}) * y\right), {\beta_D}^2(0)\} = {\beta_D}^2\left((x * y^{n+1}) * y\right), \text{ for all } x, y \in X.$$

It follows from theorem 3.18 that X_N is an (3, 2)- negative fuzzy AB-ideal of X.

Conclusion: we proposed negative implicative ideals in AB-algebras. Relationship between (3, 2)- uncertainty ideals and (3,2)- uncertainty structure negative implicative ideals are discussed. Conditions and characterization of (3, 2)- uncertainty negative implicative ideals are provided. Further researchers may do their research work in the field of various fuzzy algebraic structures.

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