https://doi.org/10.5281/zenodo.17831713

T-NORM UNCERTAINTY SOFT MODULO GROUP WITH RESPECT TO INCLUSION OF SOFT SETS

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Abstract: In this paper, we introduce the notion of T-fuzzy soft N-subgroups by using Molodtsov's definition of soft sets and investigate their related properties with respect to α-inclusion of soft sets.

Keywords: soft set, fuzzy soft set, soft N- group, complete, T- fuzzy soft N-group, α-inclusion.

Section-1: Introduction

In 1999, Molodtsov's [10] proposed an approach for modeling, vagueness and uncertainty, called soft set theory. Since its inception, works on soft set theory have been progressing rapidly with a wide-range applications especially in the mean of algebraic structures as in [1-14]. The structures of soft sets, operations of soft sets and some related concepts have been studied by [10-13].Agun and Sezgin [3] defined soft N- subgroups and soft N-ideals of an N-group. They studied their properties with respect to soft set operations in more detail. In this paper, the notion of T- fuzzy soft N-subgroups by using Molodtsov's definition of soft sets are discussed and investigate their related properties with respect to α-inclusion of soft sets.

Section-2 Preliminaries

This section contains some basic definition and preliminary results which will be needed in the sequal.In what follows let G and S denote a group and max-norm respectively unless otherwise specified.

Definition 2.1 : By a near ring, we shall mean an algebraic system (N,+,•), where

(i) (N,+) forms a group (not necessarily abelian)

(ii) (N,•) forms a semi group and

(iii) (a+b) c = ac + bc for all a,b in N.

Throughout this paper, N will always denote a right near ring whose zero element in 0_N.A subgroup M or N write N is contained in M is called a sub near ring of N. For a near ring N, the zero symmetric part of N denoted by N_O is defined by $N_0 = \{ n \, \varepsilon \, N / nN_o = O_N\}$

Definition 2.2: Let (G,+) be a group and A: N×G→G, (n,g) → ng , (G,A) is called an N-group if for all x,y ε N, for all g ε G,

(i) x(yg) = (xy)g and

(ii) (x+y)g = xg +yg. It is denoted by N^G.

Clearly, N itself is an N-group by natural operation. A subgroup H of G with NH contained in H is said to be an N-subgroup of G. Let N be a near-ring and G and ψ two N-groups. Then f: G→ ψ is called N-homomorphism if for all g,H ε G, for all n ε N.

(i) f(g+H)= f(g)+ f(H) and

(ii) f(ng) = nf(g). For all undefined concepts and notions, we refer [17].

From now on, U refers to an initial universe, E is a set of parameters, 2^U is the power set of U and A,B,C is subset of E.

Definition 2.3: Let X be a set. Then a mapping μ: X→ [0, 1] is called fuzzy subset of X.

Definition 2.4: Let U be a universal set, E set of parameters and A is a subset of E. Then a pair (F, A) is called soft set over U, where F is a mapping from A to 2^U, the power set of U.

Example 2.5: Let X = $\{c_1,c_2,c_3\}$ be the set of three cars and E= {costly (e_1), metallic colour (e_2), cheap(e_3)} be the set of parameters, where A = $\{e_1,e_2\}$ is a subset of E. Then

(F,A) = $\{F(e_1) = \{c_1,c_2,c_3\}, F(e_2) = \{c_1, c_2\}$ is crisp soft set over X.

Definition 2.6: Let U be a universal set, E set of parameters and ACE. Let F(X) denotes the set of all fuzzy subsets of U. Then a pair (F, A) is called soft set over U, where F is a mapping from A to F(U).

Example 2.7: Let U = {c_1,c_2,c_3} be the set of three cars and E= {costly (e_1), metallic colour (e_2), cheap(e_3)} be the set of parameters, where A = {e_1,e_2} C E. Then

(F,A) = {F(e_1) = {c_1/ 0.5,c_2 /0.6,c_3/0.2}, F(e_2) = {c_1/0.4, c_2/0.5, C_3/ 0.7} is the fuzzy soft set over U denoted by F_A.

Definition 2.8: Let F_A be a fuzzy soft set over U and α be a subset of U. Then upper α- inclusion of F_A denoted by

$F_A^{+\alpha}$ ={ xε A / F(x) ≥ α} similarly

$F_A^{-\alpha}$ ={ x εA / F(x) ≤ α} is called lower α-inclusion of F_A.

Definition 2.9: A triangular cnorm (*t*-norm) is a mapping T : [0, 1] × [0, 1] −→ [0, 1] that satisfies the following conditions:

(S1) T(x, 0) = x,

(S2) T(x, y) = S(y, x),

(S3) T(x, S(y, z)) = S(S(x, y), z),

(S4) T(x, y) ≤ S(x, z) whenever y ≤ z,

for all x, y, z ∈ [0, 1].

Replacing 0 by 1 in condition T, we obtain the concept of *t*-norm T.

Lemma 2.10:Let 'T' be a t- norm. Then t- norm 'T' can be defined as T(x,y) = 1- S(1-x, 1-y).

Proof: straight forward

Definition 2.11: Let F_A and K_B be fuzzy soft sets over the common universe U and ψ : A → B a function. Then fuzzy soft image of F_A under ψ over U denoted by ψ(F_A)is a set-valued function, when ψ(F_A) : B→ 2^U defined by

$$\psi(F_A)(b) = \begin{cases} T\{ F(a) / a\varepsilon A \text{ and } \psi(a) = b \}, \text{ if } \psi^{-1}(b) = \Phi \\ \Phi \quad \text{otherwise} \end{cases}$$

For all bε B. The fuzzy soft pre image of K_B under ψ over U, denoted by $\psi^{-1}(G_B)$ is a set valued function where $\psi^{-1}(G_B)$: A→ 2^U defined by $\psi^{-1}(K_B)(b)$ = G(ψ(a)) for all aε A. Then fuzzy soft anti image of F_A under ε over U denoted by ψ*(F_A) is a set valued function, where

$$\psi^*(F) = \begin{cases} S\,\{F(a) / a\ \varepsilon\ A \text{ and } \psi(a) = b\}, \text{ if } \psi^{-1}(b) = \Phi \\ \Phi \quad \text{otherwise} \quad \text{for all } b\varepsilon\ B. \end{cases}$$

Definition 2.12: Let H be an N- subgroup of G and F_H be a fuzzy soft set over G. If for all x,yε H and n ε N,

(i) $F_H(x-y) \geq T\{F_H(x), F_H(y)\}$ and

(ii) $F_H(nx) \geq F_H(x)$, then the fuzzy soft set F_H is called a fuzzy soft N-subgroup of G.

Definition 2.13: Let H be an N- subgroup of G and F_H be a fuzzy soft set over G. If for all x,yε H and n ε N,

(i) $F_H(x-y) \leq S\{F_H(x), F_H(y)\}$ and

(ii) $F_H(nx) \leq F_H(x)$, then the fuzzy soft set F_H is called T- fuzzy soft N-subgroup of G. It is denoted by F_H⌂$_N$ G

Example 2.14: Consider N = {0,1,2,3} be a near-ring with operations + and •

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

•	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	1	2
3	0	3	2	1

https://doi.org/10.5281/zenodo.17831713

Let G= N, H= {0,2}⌂N G and F_H be a fuzzy soft set over G, where F: H→ 2^G is a set valued function defined by F(x) = {0} Ụ {y ε G / 3x=y} for x ε H. Then F(0) = {0} and F(2) = {0,2}. Therefore F_H ⌂N G. If we define a fuzzy soft set K_H over G by K(x) = {y ε G / 3x=y} for all xε H. Then K(0) = {0} and K(2)= {2}.since K(2-2) = K(0) not in K(2). K_H is not a T- fuzzy soft N-subgroup of G.

Definition 2.15: The relative complement of the fuzzy soft set F_A over G is denoted by F_A^r, where $F_A^r : A \rightarrow 2^U$ is a mapping given as $F_A^r(x) = G / F_A(x)$, for all x ε A.

Section-3 Properties of Intersection uncertainty soft N-subgroup

Proposition 3.1: Let F_H be a fuzzy soft set over G and α be a subset of G. If F_H is T- fuzzy soft N-subgroup of G, then lower α-inclusion of F_H is an N-subgroup of G.

Proof: since F_H is T- fuzzy soft N-subgroup of G.

Assume x,y ε $F_H^{-\alpha}$ and n ε N, then $F_H(x) \leq \alpha$ and $F_H(y) \leq \alpha$. We need to show that x-y ε $F_H^{-\alpha}$ and n ε $F_H^{-\alpha}$. Since F_A is T- fuzzy soft N-subgroup of G, it follows that $F_H(x-y) \geq T\ \{F_H(x), F_H(y)\} \geq T\ \{\alpha,\alpha\} \geq \alpha$ and $F_H(nx) \geq F_H(x) \geq \alpha$ which completes the proof.

Proposition 3.2: Let F_H be a fuzzy soft set over G and α be a subset of G. If F_H is fuzzy soft N-subgroup of G, then upper α-inclusion of F_H is an N-subgroup of G.

Proof: since F_H is T-fuzzy soft N-subgroup of G.

Assume x,y ε $F_H^{+\alpha}$ and n N, then $F_H(x) \geq \alpha$ and $F_H(y) \geq \alpha$. We need to show that x-y ε $F_H^{+\alpha}$ and n ε $F_H^{+\alpha}$. Since F_H is fuzzy soft N-subgroup of G, it follows that $F_H(x-y) \geq T\ \{F_H(x), F_H(y)\} \geq T\ \{\alpha,\alpha\} \geq \alpha$ and $F_H(nx) \geq F_H(x) \geq \alpha$ which completes the proof.

Proposition 3.3: Let F_H be a fuzzy soft set over G.Then F_H is T- fuzzy soft N- subgroup of G if F_H^r is S-fuzzy soft N-subgroup of G.

Proof: Let F_H be a T- fuzzy soft N-subgroup of G, then, for all x,y ε H and n ε N.

$F_H^r(x-y) = G / F_H(x-y) \geq (G / D\{F_H(x), F_H(y)\}) = T\{(G/ F_H(x)), (G /F_H(y))\} = T\{F_H^r(x), F_H^r(y)\}$

$F_H^r(nx) = G / F_H(nx) \geq (G / F_H(x)) = F_H^r(x)$

F_H^r is S-fuzzy soft N-subgroup of G.

Proposition 3.4 : Let F_H: $X \rightarrow X^1$ be a soft homomorphism of N-subgroups. If F_H^f is T- fuzzy soft N-subgroups of X, then F_H is T- fuzzy soft N-subgroup of X^1.

Proof: Suppose F_H is T- fuzzy soft N-subgroups of X^1, then

(i) Let $x^1, y^1 \varepsilon X^1$, there exists x,y ε X such that $f(x) = x^1$ and $f(y) = y^1$. We have
$F_H(x^1 - y^1) = F_H(f(x) - f(y)\} \geq T\{F_H(x), F_H(y)\} = T\{F_H^f(x), F_H^f(y)\}$ and

(ii) $F_H(nx^1) = F_H(nf(x)) \geq F_H(f(x)) = F_H^f(x)$.
Therefore F_H is T- fuzzy soft N-subgroups of X^1.

Proposition 3.5: Let F_H be T- fuzzy soft N-subgroups of X and F_H^* be a fuzzy soft in X given by $F_H^*(x) = F_H(x) + 1 - F_H(1)$ for all xε X. Then F_H^* is T- fuzzy soft N-subgroups of X and F_H C F_H^*.

Proof: Since F_H is T- fuzzy soft N-subgroups of X and $F_H^*(x) = F_H(x) + 1 - F_H(x)$ for all x ε X. For any x,y ε X,we have $F_H^*(x) = F_H(1) + 1 - F_H(1) = 1 > F_H^*(x)$ and

(i) For all x,y ε X, we have

$$\begin{aligned} F_H^*(x-y) &= F_H(x-y) + 1 - F_H(1) \\ &\geq T\{F_H(x), F_H(y)\} + 1 - F_H(1) \\ &= T\{F_H(x) + 1 - F_H(1), F_H(y) + 1 - F_H(1)\} \\ &= T\{F_H^*(x), F_H^*(y)\} \end{aligned}$$

(ii)

$$\begin{aligned} F_H^*(x) &= F_H(nx) + 1 - F_H(1) \\ &\geq F_H(x) + 1 - F_H(1) = F_H^*(x) \end{aligned}$$

Therefore F_H^* is T- fuzzy soft N-subgroup of X and F_H C F_H^*.

Proposition 3.6: Let F_H and F_Δ be fuzzy soft sets over G. where H and Δ are N-subgroups of G and $\psi : H \rightarrow \Delta$ is an N-homomorphism. If F_H is T- fuzzy N-subgroups of G, then so is $\psi(F_H)$.

Proof: Let α_1, α_2 ε Δ such ψ is surjective, there exists a_1, a_2 ε H such that $\psi(a_1) = \alpha_1$ and $\psi(a_2) = \alpha_2$. Thus

(Ψf_H) (x) = T { F(H) / H ε H, $\psi(H) = \alpha_1 - \alpha_2$ }

= T { F(H) / H ε H, $H = \psi^{-1}(\alpha_1 - \alpha_2)$ }

= T { F(H) / H ε H, $H = \psi^{-1}(\psi(\alpha_1 - \alpha_2)) = A_1 - A_2$ }

= T{ $F(a_1 - a_2) / \alpha_1, \alpha_2$ ε Δ, $\psi(H_i) = \alpha_i$, i= 1,2 }

= T{ (max{$F(a_1) / \alpha_1$ ε Δ, $\psi(H_1) = \alpha_1$}), (max{$F(a_2) / \alpha_2$ ε Δ, $\psi(H_2) = \alpha_2$}),

= T { $\psi(F_H)(a_1)$, $\psi(F_H)(a_2)$}

Now let n ε N and α ε Δ. Since ψ is surjective, there exists Ĥ ε H such that ψ (Ĥ) = 0

$(\psi(F_H)(n\alpha)$ = T{ F(H) / H ε H, ψ (H) = nα }

= T { F(H) / H ε H, $H = \psi^{-1}(n\alpha)$ }

= T { F(H) / H ε H, $H = \psi^{-1}(n\,\psi(\hat{H})$ }

= T { F(H) / H ε H, $H = \psi^{-1}(\psi(nH) = n\hat{H}$ }

= T { $F(n\hat{H}) / \hat{H}$ ε H, $H = \psi^{-1}(\hat{H}) = \alpha$ }

= T { $F(\hat{H}) / \hat{H}$ ε H, $H = \psi^{-1}(\hat{H}) = \alpha$ }

= $(\psi(F_H))(\alpha)$

$\psi(F_H)$ is T- fuzzy soft N-subgroup of G.

Proposition 3.7: Let F_H: X→Y be a soft homomorphism of N-subgroups. If F_H is T- fuzzy soft N-subgroups of Y, the F_H^f is T- fuzzy soft N-subgroups of X.

Proof:Suppose F_H is T- fuzzy soft N-subgroups of Y, then for all

(i) For all x,y ε X, we have
$F_H^f(x-y) = F_H(f(x) - f(y)\} \geq T\{F_H(f(x)), F_H(f(y))\} = T\{F_H^f(x), F_H^f(y)\}$ and
(ii) $F_H^f(nx) = F_H(nf(x)) \geq F_H(f(x)) = F_H^f(x)$.
Therefore F_H is intersection fuzzy soft N-subgroups of Y..

Proposition 3.8: Let F_H and F_Δ be fuzzy soft sets over G, where H and Δ are N-subgroups of G and ψ be an N-homomorphism from H to Δ. If F_Δ is T- fuzzy soft N- subgroups of G, then so is $\psi^{-1}(F_\Delta)$.

Proof: Let a_1,a_2 ε H, then

$$(\psi^{-1}(F_\Delta)(a_1-a_2) = F(\psi\ (a_1-a_2))$$

$$\geq T\{F(\psi\ (a_1), (\psi\ (a_2)\}$$

$$\geq T\{\ \psi^{-1}(F_\Delta)(a_1), \psi^{-1}(F_\Delta)(a_2)\}$$

Now let n ε N and H ε H, then

$$(\psi^{-1}(F_\Delta)(nH) = F(\psi\ (nH)) = F(n\ \psi\ (H)) = G(\psi\ (H)) = \psi^{-1}(F_\Delta)(H)$$

Therefore $\psi^{-1}(F_\Delta)$ is T- fuzzy soft N-subgroups of G.

Proposition 3.9: A fuzzy soft subset 'F_H' of G is S- fuzzy soft N-subgroups of G.

if and only if 'F_H^C' is . T- fuzzy soft N-subgroups of G.

Proof: Let ''FH be a 'T- fuzzy soft N-subgroups of G. for all x,y ε G, we have

(i) $FH^c(x-y) = 1- F_H(x-y)$

$$\leq 1- T(F_H(x), F_H(y))$$

$$= 1- T(1- F_H^c(x), 1- F_H^c(y))$$

$$= S(F_H^c(x), F_H^c(y))$$

(ii) $FH^c(nx) = 1- F_H(nx)$

$$\leq 1- F_H(x) = F_H^c(x)$$

F_H^c is T- fuzzy soft N-subgroups of G.

Proposition 3.10: If F_H and F_Δ be two T-fuzzy soft N-subgroups of G, then $F_H Ụ F_\Delta$ also T-fuzzy soft N-subgroup of G

Proof: Since H and Δ are N-subgroup of G, then H∩Δ is an N-subgroup of G. Let P = F_H ỤF_Δ. , where P(x) = $F_H(x)$ Ụ $F_\Delta(x)$ for all x ε H∩Δ not equal to empty. Then for all x,y ε H∩Δ and n ε N,

P(x-y) = $F_H(x-y)$ Ụ $F_\Delta(x-y)$

≥ T {T{ $F_H(x)$, $F_H(y)$}, T{$F_\Delta(x)$, $F_\Delta(y)$}}

= T{ T{$F_H(x)$, $F_\Delta(x)$}, T{$F_H(y)$, $F_\Delta(y)$}}

= T{ P(x), P(y)}

P(nx) = $F_H(nx)$ Ụ $F_\Delta(nx)$

≥ T{$F_H(x)$, $F_\Delta(x)$}

= P(x)

Therefore F_H Ụ F_Δ also T-fuzzy soft N-subgroup of G.

Definition 3.1: A T-fuzzy soft N-subgroup FH of G is said to be complete if it is normal and there exists x ε X such that $F_H(z) = 0$.

Proposition 3.11: Let F_H be T-fuzzy soft N-subgroup of G and let w be a fixed element of G such that $F_H(1) = F_H(w)$. Define a fuzzy soft set F_H* in G by F_H*(x) = $F_H(x) – F_H(w) / F_H(1) – F_H(w)$ for all x G. Then F_H* is complete T-fuzzy soft N-subgroup of G.

Proof: For any x,y G , we have

F_H*(x-y) = $F_H(x-y) – F_H(x) / F_H(1)- F_H(w)$

≥ T {$F_H(x)$, $F_H(y)$} – $F_H(w)$ / $F_H(1)$- $F_H(w)$

= T {{$F_H(x)$, $F_H(y)$} – $F_H(w)$ / $F_H(1)$- $F_H(w)$}, {$F_H(x)$, $F_H(y)$} – $F_H(w)$ / $F_H(1)$- $F_H(w)$}}

= T { F_H*(x), F_H*(y)}

$F_H^*(nx) = F_H(nx) – F_H(x) / F_H(1)- F_H(w)$

$\geq F_H(x), F_H(y)\} – F_H(w) / F_H(1)- F_H(w)\}$

$= F_H^*(x)$ Therefore F_H^* is an complete T-fuzzy soft N-subgroup of G.

Conclusion: This paper summarized the basic concepts of fuzzy soft sets. By using these concepts, we studied the algebraic properties of T- fuzzy soft N-subgroups. This work focused on fuzzy pre-image, fuzzy soft image, fuzzy soft anti-image. To extend this work, one could study the properties of M-fuzzy soft N-subgroups in other algebraic structures such as rings and fields.

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Received: Nov 01, 2025 **Accepted:** Dec 01, 2025 **Published:** Dec 05, 2025